ANSWER KEY

Honors 125
Problems on limits

Due Wednesday, October 21, 2009

Find the following limits, if they exist. If not, write DNE. If you cannot find a solution algebraically, use a calculator and a table of values.

1. $\lim_{x\to-3}\frac{x+3}{x^2+4x+3}$ $= \lim_{x\to-3}\frac{\cancel{x+3}}{(\cancel{x+3})(x+1)} = \lim_{x\to-3}\frac{1}{x+1} = \boxed{\frac{1}{-2} = -\frac{1}{2}}$

plug in: $\frac{-3+3}{(-3)^2+4(-3)+3} = \frac{0}{0}$: keep going

2. $\lim_{x\to2}\frac{x+1}{x-2}$: $\boxed{DNE}$ $\left(\frac{\text{non-zero constant}}{0}\right)$

plug in: $\frac{3}{0}$

3. $\lim_{x\to-1}\frac{x+1}{x^2-1}$ $= \lim_{x\to-1}\frac{\cancel{(x+1)}}{(\cancel{x+1})(x-1)} = \lim_{x\to-1}\frac{1}{x-1} = \boxed{\frac{1}{-2} = -\frac{1}{2}}$

plug in: $\frac{-1+1}{(-1)^2-1} = \frac{0}{0}$: keep going

4. $\lim_{x\to-1}\frac{x^2-1}{x^2-2x+1}$ $= \boxed{0}$

plugin: $\frac{(-1)^2-1}{(-1)^2-2(-1)+1} = \frac{0}{4} = 0$

You will need tables of values for the following four problems:

5. $\lim_{x\to0}\frac{2e^{3x}-2}{x}$ $= \boxed{6}$ (from table)

x	$-.1$	$-.01$	$-.001$	$-.0001$	0	$.0001$	$.001$	$.01$	$.1$
$\frac{2e^{2x}-2}{x}$	5.1836	5.91089	5.9910	5.9991	—	6.00090	6.00901	6.09091	6.99718

$\rightarrow 6 \leftarrow$

plugin: $\frac{2e^{3\cdot0}-2}{0} = \frac{2\cdot1-2}{0} = \frac{0}{0}$: keep going

6. $\lim_{x\to0}\frac{|x|}{x}$ (Be sure to check the limit from both above and below $x = 0$.) (plug in: $\frac{0}{0}$: keep going)

$\boxed{\lim_{x\to0}\frac{|x|}{x} \text{ DNE}}$ because $\lim_{x\to0^-}\frac{|x|}{x} \neq \lim_{x\to0^+}\frac{|x|}{x}$

x	$-.1$	$-.01$	$-.001$	$-.0001$	0	$.0001$	$.001$	$.01$	$.1$
$\frac{\lvert x\rvert}{x}$	-1	-1	-1	-1	—	1	1	1	1

7. $\lim_{x\to0^+}x^x$ $= 1$ (from table)

x	$.0001$	$.001$	$.01$	$.1$
	.99908	.99312	.95499	.79433

$1 \leftarrow$

8. $\lim_{x\to1}\frac{x^5-1}{x-1}$ (plugin: $\frac{0}{0}$) $= 5$ (from table)

x	$.9$	$.99$	$.999$	$.9999$	1	1.0001	1.001	1.01	1.1
$\frac{x^5-1}{x-1}$	4.0951	4.90995	4.99001	4.9990	—	5.00100	5.01001	5.101..	6.1051

$\rightarrow 5 \leftarrow$

9. $\lim_{x\to\infty}\frac{x^2-1}{3x^4-7x^2}$ $=$

$$\lim_{x\to\infty}\frac{\frac{x^2}{x^4}-\frac{1}{x^4}}{\frac{3x^4}{x^4}-\frac{7x^2}{x^4}} = \lim_{x\to\infty}\frac{\frac{1}{x^2}-\frac{1}{x^4}}{3-\frac{7}{x^2}} = \frac{\lim_{x\to\infty}\frac{1}{x^2}-\lim_{x\to\infty}\frac{1}{x^4}}{\lim_{x\to\infty}3-\lim_{x\to\infty}\frac{7}{x^2}} = \frac{0-0}{3-0} = \frac{0}{3} = \boxed{0}$$

10. Section 3.3, #30, 32, 34

#30. $$\lim_{x\to+\infty}\frac{x^6+3000x^3+1{,}000{,}000}{2x^6+1000x^3} = \lim_{x\to\infty}\frac{\frac{x^6}{x^6}+\frac{3000x^3}{x^6}+\frac{1{,}000{,}000}{x^6}}{\frac{2x^6}{x^6}+\frac{1000x^3}{x^6}} = \lim_{x\to\infty}\frac{1+\frac{1000}{x^3}+\frac{1{,}000{,}000}{x^6}}{2+\frac{1000}{x^3}} = \frac{1+0+0}{2+0} = \boxed{\frac{1}{2}}$$

32. $$\lim_{x\to+\infty}\frac{2x^4+20x^3}{1000x^3+6} = \lim_{x\to+\infty}\frac{\frac{2x^4}{x^3}+\frac{20x^3}{x^3}}{\frac{1000x^3}{x^3}+\frac{6}{x^3}} = \lim_{x\to+\infty}\frac{2x+20}{1000+\frac{6}{x^3}} \to +\infty \quad \boxed{\text{(limit DNE)}}$$

34. $\lim_{x\to+\infty}\frac{2x^4+3x^3}{1000x^6+6} = 0$ (highest exponent is in denominator)

11. Let $f(x)=\begin{cases} 6x-2, \text{ if } x<1 \\ 7, \text{ if } x=1 \\ -x+5, \text{ if } x>1 \end{cases}$

a) Find $\lim_{x\to 3} f(x)$, if it exists. use $f(x) = -x+5$. $\lim_{x\to 3} -x+5 = -3+5 = \boxed{2}$

b) Find $\lim_{x\to 1^+} f(x)$, if it exists. $= \lim_{x\to 1^+} -x+5 = -1+5 = \boxed{4}$

c) Find $\lim_{x\to 1^-} f(x)$, if it exists. $= \lim_{x\to 1^-} 6x-2 = 6-2 = \boxed{4}$

d) Find $\lim_{x\to 1} f(x)$, if it exists. $\lim_{x\to 1} f(x) = \boxed{4} = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

e) Find $f(1)$, if it exists. $= \boxed{7}$

Note: the function is NOT continuous at $x=1$, because even though the limit exists, the function value does not equal the limit (the "hole" is not filled in).