Honors 125
Test 1

Name ANSWER KEY
September 28, 2009

You must show all work, and your work must be neat, to receive credit for your answers.

6 1. Find the equation, in slope-intercept form, of the line through the points (-2, 1) and (2, 3).

$m = \frac{3-1}{2-(-2)} = \frac{2}{4} = 1/2$ $y - 1 = 1/2(x+2) = \frac{1}{2}x + 1$

$\boxed{y = 1/2 x + 2}$

2. Consider the following chart of values.

Difference Ratio Difference Ratio Difference

x	$F(x)$	$G(x)$	$H(x)$
-1	8	8	36
0	12	12	18
1	18	16	9
2	27	20	4.5
3	40.5	24	2.25

exp. Linear

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6 a) Which set(s) of values, if any, could represent a linear function? G(x) Briefly justify your answer and write an equation for each linear function.

- Differences between successive terms are constant (= 4). None of the other functions exhibit that.
- $G(x) = 4x + 12$

12 b) Which set(s) of values, if any, could represent an exponential function? F(x) and H(x) Briefly justify your answer and write an equation for each exponential function.

Both have constant ratios of later/earlier terms; for F(x) the ratio is 1.5; for H(x) the ratio is 1/2. $F(x) = 12(1.5)^x$; $H(x) = 18(1/2)^x$

c) If any set of values is neither exponential nor linear, explain why. None

3. Let $f(x) = 2x^2 - 4x - 3$.

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3 a) Does the graph of the function open up or down? up Briefly justify your answer. f(x) is quadratic; its graph is a parabola; opens up because the leading coefficient is positive (ie, a = 2, so a > 0).

4 b) What is the vertex (both coordinates) of the graph? (1, -5)

$x = -\frac{b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$; $y = (2)(1^2) - 4(1) - 3 = 2 - 4 - 3 = 2 - 7 = -5$

2 c) What is the y-intercept? $y = -3$ (or (0, -3))

4 d) What are the x-intercepts, if any? $\frac{4 \pm \sqrt{40}}{4}$ $\left(= \frac{2 \pm \sqrt{10}}{2}\right)$

Let $2x^2 - 4x - 3 = 0$ Doesn't factor; use quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$

$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2 \cdot 2} = \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4} = \frac{4 \pm \sqrt{4}\sqrt{10}}{4} = \frac{4 \pm 2\sqrt{10}}{4} = \frac{2 \pm \sqrt{10}}{2}$

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4. Find the equation of the exponential function through the points (1, 4) and (4, 2). $f(x) = A \cdot b^x$

$4 = A \cdot b^1$ $\qquad$ $2 = A \cdot b^4$ $\qquad$ $b = \sqrt[3]{1/2}$ $\qquad$ $\boxed{f(x) = \frac{4}{\sqrt[3]{1/2}} \cdot (\sqrt[3]{1/2})^x}$

$A = \frac{4}{b}$ $\qquad$ $2 = \frac{4}{b} \cdot b^4 = 4b^3$ $\qquad$ $A = \frac{4}{\sqrt[3]{1/2}}$

$b^3 = 2/4 = 1/2$

5. The amount, in mg, of a sample of Cesium-137 that is left after a period of t years is given by the function $Q(t) = 100 \cdot (0.9772)^t$.

a) How much Cesium-137 was contained in the original sample? 100 mg.

3

b) Is the amount of Cesium-137 increasing or decreasing? decreasing Briefly justify your answer. $b = 0.9772$, which is less than 1.

4

c) What is the annual percentage change in the amount of Cesium-137? -2.28% (or 2.28% decrease)

$.9772 - 1 = -.0228 = -2.28\%$

3

6. A manufacturer of surfboards determines that he sells about 100 surfboards per month when the boards are priced at \$550 each, and that he sells about 110 boards when they go on sale for \$525 each.

a) Write a linear demand function in terms of p, the price of a surfboard, that models the sales of surfboards. Points (550, 100); (525, 110) $\boxed{q(p) = -2/5\, p + 320}$

slope $= m = \frac{100-110}{550-525} = \frac{-10}{25} = -\frac{2}{5}$. $\qquad$ $100 = -2/5\,(550) + b$

3 $\qquad$ $100 = -220 + b$

$b = 320$

b) Write a function that models the manufacturer's monthly revenue from the sale of surfboards.

3

$R(p) = p \cdot q = p(-2/5\,p + 320) = \boxed{-2/5\,p^2 + 320p = R(p)}$

c) At what selling price would the manufacturer maximize his revenue?

3

at the vertex: $\frac{-b}{2a} = \frac{-320}{2(-2/5)} = \frac{320}{4/5} = \frac{320 \cdot 5}{4} = \boxed{\$400}$

d) What is the maximum revenue the manufacturer can receive from the sale of surfboards?

3

$R(400) = 400(-2/5\,(400) + 320) = 400\,(160) = \$64{,}000$

7. A colony of bacteria starts with 200 cells and triples in size every 4 hours.

4

a) Find a function that models the population of bacteria t hours after the start.

$600 = 200 \cdot b^4$ $\qquad$ $3 = b^4$ $\qquad$ $b = \sqrt[4]{3}$; $P(t) = 200 \cdot (\sqrt[4]{3})^t = 200 \cdot 3^{t/4}$

P_0

b) How many bacteria will there be after 6 hours?

3 $Q(6) = 200 \cdot 3^{6/4} = 200 \cdot 3^{3/2} \approx 1039.2 \approx 1039$ bacteria.

8. The number of passengers who used the Short Line Railroad during 1950 was 56,243. The number who used it in 1951 was 55,617.

   a) Find a linear model for the number of passengers using the Short Line t years after 1950, if the decline continued at the same number of passengers per year.

3

$P(t) = 56{,}243 - 626t$

$$\begin{array}{r} 56{,}243 \\ -\,55{,}617 \\ \hline 626 \end{array}$$

   b) What does the slope of the linear model represent? - the railroad would have 626 fewer passengers per year. (rate of decline is 626 passengers per year.)

2

   c) Find an exponential model for the number of passengers using the Short Line t years after 1950, if the percentage rate of decline continued unchanged.

3

$\frac{55617}{56{,}243} = b$ $\quad P(t) = 56{,}243 \cdot \left(\frac{55{,}617}{56{,}243}\right)^t \doteq 56{,}243\,(0.9889)^t$

   d) What was the expected number of passengers in 1960, using the linear model?

✓

49,983 passengers $\quad t=10$

$56{,}243 - 626(10) = 56{,}243 - 6260 = 49{,}983$

   e) What was the expected number of passengers in 1960, using the exponential model?

✓

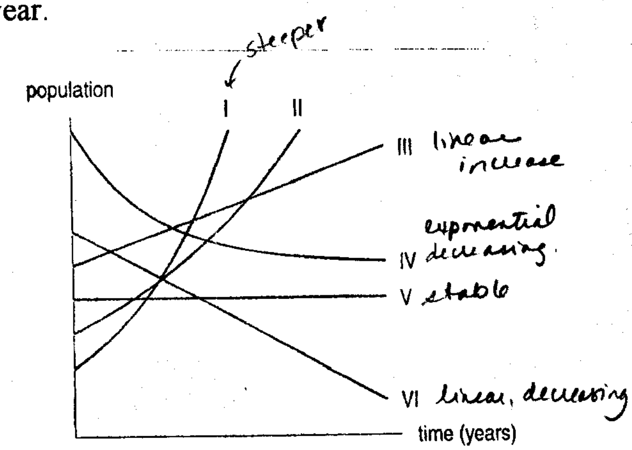
50,287 passengers $\quad t=10$

$56{,}243 \cdot \left(\frac{55617}{56243}\right)^{10} = 50{,}287.4$

other decimal approximations will give slightly different numbers for P.

9. The figure below shows graphs of several cities' populations against time. First match each of the following descriptions to a graph. Then write a description to match each of the remaining graphs.

   a) __II__ The population increased at 5% per year.
   b) __I__ The population increased at 8% per year.
   c) __III__ The population increased by 5000 people per year.
   d) __V__ The population was stable.



IV. The population decreased by 7% per year.

VI : The population decreased by 7500 people per year