Honors 125
Test 1

Name ANSWER KEY
September 28, 2009

You must show all work, and your work must be neat, to receive credit for your answers.

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1. Find the equation, in slope-intercept form, of the line through the points (-3, 2) and (1, 4).

$m = \frac{4-2}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$; $y - 2 = \frac{1}{2}(x+3) = \frac{1}{2}x + \frac{3}{2}$; $\boxed{y = \frac{1}{2}x + \frac{7}{2}}$ $= \frac{1}{2}x + 3\frac{1}{2}$

2. Consider the following chart of values.

ratio, difference, ratio, difference, ratio

x	f(x)	g(x)	h(x)
-1	8	24	24
0	12	12	20
1	18	6	16
2	27	3	12
3	40.5	1.5	8

exponential, linear

6 pts.

a) Which set(s) of values, if any, could represent a linear function? $h(x)$ Briefly justify your answer and write an equation for each linear function.

$h(x) = -4x + 20$ (the difference between successive terms is constant: later - earlier = -4.)

12 pts.

b) Which set(s) of values, if any, could represent an exponential function? $f(x)$ and $g(x)$ Briefly justify your answer and write an equation for each exponential function.

$f(x) = 12 \cdot (1.5)^x$ ← constant ratio $\left(\frac{\text{later}}{\text{earlier}}\right) = 1.5$

$g(x) = 12 \cdot (\frac{1}{2})^x$ ← constant ratio $\left(\frac{\text{later}}{\text{earlier}}\right) = \frac{1}{2}$

c) If any set of values is neither exponential nor linear, explain why. NONE

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3. Let $f(x) = 3x^2 - 6x - 2$.

3

a) Does the graph of the function open up or down? up Briefly justify your answer. Leading coefficient is positive ($a = 3$, and $3 > 0$).

4

b) What is the vertex (both coordinates) of the graph? $(1, -5)$

$x = \frac{-b}{2a} = \frac{-(-6)}{2 \cdot 3} = \frac{6}{6} = 1$ $\quad f(1) = 3(1)^2 - 6(1) - 2 = 3 - 6 - 2 = -5$

2

c) What is the y-intercept? $y = -2$ (or $(0, -2)$)

4

d) What are the x-intercepts, if any? $\boxed{\frac{6 \pm \sqrt{60}}{6}} = \frac{3 \pm \sqrt{15}}{3}$

let $3x^2 - 6x - 2 = 0$. use quadratic formula: $x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(-2)}}{2 \cdot 3} = \frac{6 \pm \sqrt{36+24}}{6}$

$= \frac{6 \pm \sqrt{60}}{6} = \frac{6 \pm 2\sqrt{15}}{6} = \frac{3 \pm \sqrt{15}}{3}$

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4. Find the equation of the exponential function through the points (1, 6) and (4, 2). use $f(x)=y=A\cdot b^x$.

$6=A\cdot b^1$ $\quad 2=A\cdot b^4$ $\quad A=\frac{6}{\sqrt[3]{1/3}}$ $\Rightarrow \boxed{f(x)=\frac{6}{\sqrt[3]{1/3}}\left(\sqrt[3]{1/3}\right)^x}$ $\approx \frac{6}{\sqrt[3]{1/3}}(.6934)^x$

$A=\frac{6}{b}$ $\quad 2=\frac{6}{b}\cdot b^4=6b^3$

$b^3=1/3$; $b=\sqrt[3]{1/3}$

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5. The amount, in mg, of a sample of Strontium-90 that is left after a period of t years is given by the function $Q(t)=500\cdot(0.9762)^t$.

3 a) How much Strontium-90 was contained in the original sample? 500 mg

4 b) Is the amount of Strontium-90 increasing or decreasing? decreasing Briefly justify your answer. $b=0.9762$, which is less than 1. Therefore this is an exponential decay function.

3 c) What is the annual percentage change in the amount of Strontium-90? decrease of 2.38%, or -2.38% change.

$.9762-1=-.0238=-2.38\%$ change

12

6. A manufacturer of washing machines determines that he sells about 200 washing machines per month when the washers are priced at \$550 each, and that he sells about 220 machines when they go on sale for \$525 each.

3 a) Write a linear demand function as a function of p, the price of a washing machine, that models the sales of washing machines. Points (550, 200) and (525, 220)

$m=\frac{220-200}{525-550}=\frac{20}{-25}=-4/5$ $\quad 200=-4/5(550)+b=-440+b$ $\quad b=640$ $\quad \therefore g(p)=-\frac{4}{5}p+640$

3 b) Write a function that models the manufacturer's monthly revenue from the sale of washing machines. $R(p)=p(-4/5p+640)=\boxed{-4/5p^2+640p=R(p)}$

3 c) At what selling price would the manufacturer maximize his revenue?

at the vertex: $p=\frac{-b}{2a}=\frac{-640}{2(-4/5)}=\frac{(640)(5)}{8}=\boxed{\$400}$

3 d) What is the maximum revenue the manufacturer can receive from the sale of washing machines?

$R(400)=400(-4/5\cdot 400+640)=400(-320+640)=400(320)=\boxed{\$128,000}$

1

7. A colony of bacteria starts with 400 cells and triples in size every 6 hours.

4 a) Find a function that models the population of bacteria t hours after the start.

$1200=400\cdot b^6 \Rightarrow 3=b^6 \Rightarrow b=\sqrt[6]{3}$ so $\boxed{P(t)=400\cdot(\sqrt[6]{3})^t}=400\cdot 3^{t/6}$

3 b) How many bacteria will there be after 8 hours?

$P(8)=400(\sqrt[6]{3})^8=400(3)^{8/6}\approx 1730.6\sim\boxed{1731 \text{ bacteria}}$

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3

8. The number of passengers who used the Dinkytown Railroad during 1960 was 68,237. The number who used it in 1961 was 66,583.

a) Find a linear model for the number of passengers using the Dinkytown t years after 1960, if the decline continued at the same number of passengers per year.

$$\begin{array}{r} 68{,}237 \\ -\,66{,}583 \\ \hline 1{,}654 \end{array}$$

$\boxed{P(t) = 68{,}237 - 1654t}$

3

b) What does the slope of the linear model represent? That each year 1654 fewer passengers ride the Railroad.

3

c) Find an exponential model for the number of passengers using the Dinkytown t years after 1960, if the percentage rate of decline continued unchanged.

$$P(t) = 68{,}237 \cdot \left(\frac{66583}{68{,}237}\right)^t = 68{,}237(.9758)^t$$

2

d) What was the expected number of passengers in 1965, using the linear model?

$\underset{t=5}{1965}$

59,967 passengers

$$P(5) = 68{,}237 - 1654(5) = 59{,}967$$

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e) What was the expected number of passengers in 1965, using the exponential model?

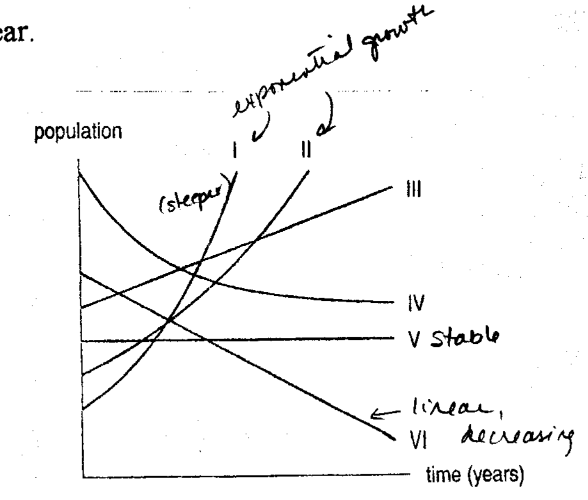
60,358 passengers

$$P(5) = 68{,}237\left(\frac{66583}{68\,237}\right)^5 \approx 68{,}237(.9758)^5 = 60{,}358.3 \approx 60{,}358$$

12 pts.

9. The figure below shows graphs of several cities' populations against time. First match each of the following descriptions to a graph. Then write a description to match each of the remaining graphs.

a) __V__ The population was stable.
b) __VI__ The population decreased by 7500 people per year.
c) __II__ The population increased at 5% per year.
d) __I__ The population increased at 8% per year.



III. The population increased linearly (by about 5000 people per year)

IV. The population decreased exponentially (by about 7%(?) per year)

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