Math 214.
Lecture 1

$x^2+2x=0$ — algebraic eqn

$x^2+2\underbrace{\frac{dx}{dt}}=0$ — differential eqn $\boxed{\text{ODE}}$ ordinary DE

$x(t)$ x', $\dot{x}$ derivative

$x^2+2\frac{\partial x}{\partial t}+2\frac{\partial x}{\partial y}=0$ - PDE — not in 214.
partial DE

$x(t,y)$

Modeling using DE.

Assumptions $\xrightarrow{\text{write DE}}$ Math formulation

↑ modify if necessary ↓ solve

check model

Analyze the Solution $\xleftarrow[\text{predictions}]{}$ Obtain Solution

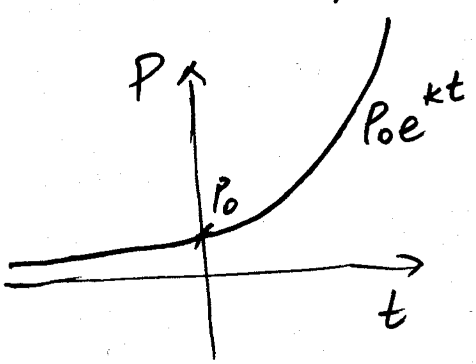
Example:

Population growth

$P(t)$ - size of mice population at time t

$\frac{dP}{dt}$ - rate of change

Assumption: $\frac{dP}{dt} \approx kP$
proportional



$$\begin{cases} \frac{dP}{dt}=kP \\ P(0)=P_0 \end{cases}$$

$P(t)=P_0 e^{kt}$
exp. growth

$\lim_{t\to+\infty} P(t)=+\infty$

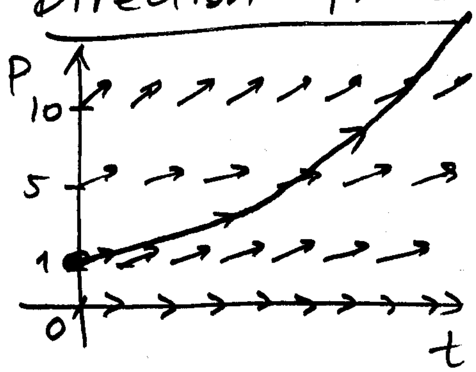
$\lim_{t\to-\infty} P(t)=0$

Modify model to include predators:

$$\frac{dP}{dt} = kP\left(1-\frac{P}{N}\right) \quad \text{logistic growth model.}$$

N – threshold, large value

Direction field ("dfield" applet)



$\frac{dP}{dt} = kP \qquad k=1$

if $P=0$ $\quad \frac{dP}{dt} = 0$ – slope $=0$

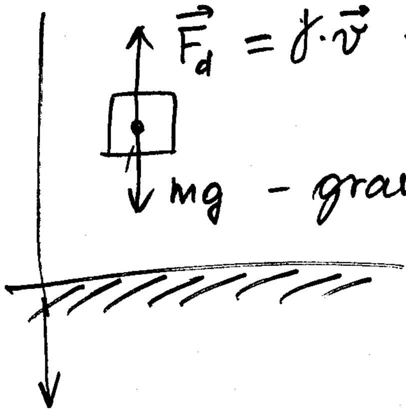
if $P=1$ $\quad \frac{dP}{dt} = 1 \leftarrow$ slope $=1$

if $P=10$ $\quad \frac{dP}{dt} = 10 \leftarrow$ slope $=10$.

Example 2: Falling body:

v – velocity

a – acceleration $= \frac{dv}{dt}$



$\vec{F}_d = \gamma \cdot \vec{v}$ – resistance

mg – gravity

$F_{net} = ma$

$F_{net} = mg - \gamma v$

$\Rightarrow mg - \gamma v = ma$

$$\boxed{mg - \gamma v = m\frac{dv}{dt}}$$

$$m\frac{dv}{dt} = mg - \gamma v$$

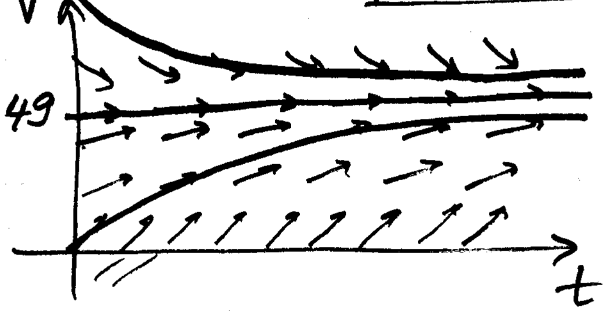
Num. values: $\quad m = 10\,[kg] \qquad g = 9.8\left[\frac{m}{sec^2}\right]$

$\gamma = 2\,\left[\frac{kg}{sec}\right]$

$$\Rightarrow \quad 10\frac{dv}{dt} = 98 - 2v$$

$$\boxed{\frac{dv}{dt} = 9.8 - 0.2v}$$

$V=49 \quad \frac{dv}{dt} = 9.8 - 9.8 = 0$



$V=0 \quad \frac{dv}{dt} = 9.8$

$V=1 \quad \frac{dv}{dt} = 9.6$

$V=40 \quad \frac{dv}{dt} = 9.8 - 8 = 1.8$

$V=50 \quad \frac{dv}{dt} = 9.8 - 10 = -0.2$

Analytical solution: Method of separation of variables.

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}$$

$$\frac{dv}{dt} = \frac{49-v}{5}$$

$$\frac{dv}{dt} = -\frac{1}{5}(v-49)$$

$$\frac{dv}{v-49} = -\frac{1}{5}dt$$ integrate both parts

$$\int \frac{dv}{v-49} = \int -\frac{1}{5}dt$$

$$\ln|v-49| = -\frac{t}{5} + C$$ ← exponentiate

$$|v-49| = e^{-\frac{t}{5}+C} = e^{-\frac{t}{5}} \cdot \underbrace{e^{C}}_{C_1}$$

$$|v-49| = C_1 e^{-\frac{t}{5}}$$

$$v-49 = \underbrace{\boxed{\pm C_1}}_{C} e^{-\frac{t}{5}}$$

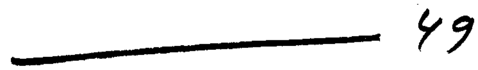
$$\left[v(t) = 49 + Ce^{-t/5}\right],$$ where C is an arbitrary const.

if $v(0) = v_0 \Rightarrow$

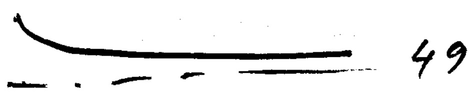
$$v(0) = 49 + C \Rightarrow C = v_0 - 49$$

$$\Rightarrow \boxed{v(t) = 49 + (v_0 - 49)e^{-t/5}}$$

if $v_0 = 49$ $\lim_{t\to\infty} v(t) = 49$



if $v_0 > 49$ $\lim_{t\to\infty} v(t) = 49$



if $v_0 < 49$ $\lim_{t\to\infty} v(t) = 49$