

**Math 685/CSI 700/OR 682 Project 1.3**  
**given 04/05/08, due in class 04/18/08**

*Each group should submit a report, which should include a short 2-5 page account on the implementation and solution, plus any necessary supplements in the form of graphs and/or tables, in class on the date listed above. Matlab codes should be sent to me via email (in archived form if necessary). Late submissions will bear a penalty.*

**Problem 3.**

The Lower Colorado River consists of a series of four reservoirs. Mass balances can be written for each reservoir resulting in the following system of linear algebraic equations:

$$\begin{bmatrix} 13.422 & 0 & 0 & 0 \\ -13.422 & 12.252 & 0 & 0 \\ 0 & -12.242 & 12.377 & 0 \\ 0 & 0 & -12.377 & 11.797 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 750.5 \\ 300 \\ 102 \\ 30 \end{bmatrix}$$

where the right-hand side vector  $f$  consists of the loadings of chloride to each of the four lakes and  $c_1, c_2, c_3, c_4$  are the resulting chloride concentrations for Lakes Powell, Mead, Mohave and Havasu, respectively.

(a) Reformulate the problem as a linear system  $Ac = f$ . Write a program to solve for concentrations in each of the lakes.

(b) Estimate  $\|A\|$  via column sums and  $\|A^{-1}\|$  via choosing  $f$  so that the ratio  $\|c\|/\|f\|$  is large, where  $c$  is the solution to  $Ac = f$  above. Compute the condition number using these estimates and compare with the value given by MATLAB condition number estimator `cond`.

(c) Analyze the problem for accuracy and stability. How many accurate digits can you expect in each calculation? Can you say something about the forward and backward errors in this calculation?

(d) Experiment with different chloride loadings and observe their effect on lake concentrations. Draw concentration in lake Havasu as a function of the loading to Lake Powell. How much must the loading to Lake Powell be reduced in order for the chloride concentration in lake Havasu to be 75?