

Math 685/CSI 700/OR 682 Homework 4
given 03/27/08

The following are some practice problems I would like you to do. Solutions will be discussed in class on 03/31/08 and 04/07/08.

Suggested reading: Heath, Chapters 4,5

Problem 1.

Consider

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}.$$

- (a) Find the characteristic polynomial of A and determine its roots. What are the eigenvalues of A and the corresponding eigenvectors?
- (b) Perform one iteration of the power iteration on A , starting with $x_0 = [1, 1]^T$.
- (c) To what eigenvector will the power iteration ultimately converge?
- (d) What eigenvalue estimate is given by the Rayleigh quotient, using the vector $x = [1, 1]^T$?
- (e) To what eigenvector of A would the inverse power iteration ultimately converge?
- (f) What eigenvalue would be obtained if inverse iteration were used with shift $\sigma = 2$?
- (g) If QR iteration were applied to A , to what form would it converge: diagonal or triangular? Why?

Problem 2.

Let A be a real symmetric tridiagonal matrix having no zero entries on its sub-diagonal. Show that A must have distinct eigenvalues.

Problem 3.

- (a) Compute the eigenvalues and eigenvectors of the matrix $A = [1, 1000; 0.001, 1]$.
- (b) Compute the condition number of the matrix of eigenvectors and also the absolute condition number of each eigenvalue.
- (c) What are the eigenvalues of the matrix $B = [1, 1000; 0, 1]$? How do the changes in the eigenvalues resulting from the perturbation of A compare with expectations?

Problem 4.

What is the convergence rate for Newton's method for finding the root $x = 2$ for each of the following equations?

- (a) $f(x) = (x - 1)(x - 2)^2 = 0$
- (b) $f(x) = (x - 1)^2(x - 2) = 0$

Write down the Newton's method iteration and explain how this can be considered a special case of the fixed point iteration $x_{k+1} = g(x_k)$ for appropriate choice of g .