Math 685/CSI 700/OR 682 Homework 3
given 02/12/08

Suggested reading: Heath, Chapters 1,2

Problem 1.
How is `cond(A)` defined for a given matrix norm? How can you use the condition number to estimate the accuracy of the computed solution to a linear system $Ax = b$? Compute the condition number for the matrix given below using 1-norm. Is your answer different when you use a ∞-norm?

$$A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Problem 2.
Suppose that both sides of an arbitrary system of linear equations $Ax = b$ is premultiplied by a nonsingular diagonal matrix. Does this change the true solution x? Does this affect the conditioning of the system? the choice of pivots in Gaussian elimination?

Problem 3.
Can every nonsingular $n \times n$ matrix A be written as a product of a lower-triangular and upper-triangular matrix $A = LU$? If yes, what is the algorithm accomplishing this? If not, give a counterexample.

Problem 4.
In solving a linear system $Ax = b$, what is meant by the residual of an approximate solution $\tilde{x}$? Does a small residual always imply that the solution is accurate? Does a large residual always imply that the solution is not accurate? Explain.

Problem 5.
Assume that you are solving a system of linear equations $Ax = b$ on a computer whose floating-point number system has 12 decimal digits of precision, and that the problem data are correct to full machine precision. About how large can the condition number of the matrix A be before the computed solution x will contain no significant digits?

Problem 6.
Prove the Sherman-Morrison formula

$$(A - uv^T)^{-1} = A^{-1} + A^{-1}u(1 - v^T A^{-1}u)^{-1}v^T A^{-1}$$

Hint: multiply both sides by $A - uv^T$. What does this formula accomplish?

Problem 6.
Use a library routine for LU decomposition (command `lu` in MATLAB) and

then perform a 2-step Gaussian elimination algorithm `Lz=b,Ux=z` to solve the system $Ax = b$, where

$$A = \begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -1 & 7 \end{pmatrix}, b = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix}.$$

Use the computed LU decomposition of A to solve the system $Ay = c$, where $c = (4, 8, -6)^T$, without refactoring the matrix.

(b)If the matrix A changes so that $a_{1,2} = 2$, use the Sherman-Morrison updating technique to compute the new solution x without refactoring the matrix and using the original right hand side vector b. How much computational effort has been saved compared to re-doing Gaussian elimination on this matrix?