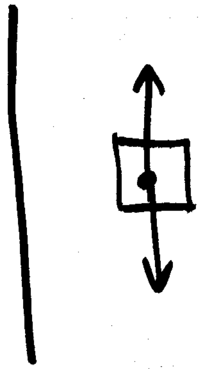
Math 214.002

Lecture 7



Falling body problem.

$$\boxed{F = ma}$$

net force

acceleration

if $v(t)$ - velocity

$\boxed{v'(t) = a(t)}$ $x'(t) = v(t)$

position.

F ~~is~~ gravity + air resistance

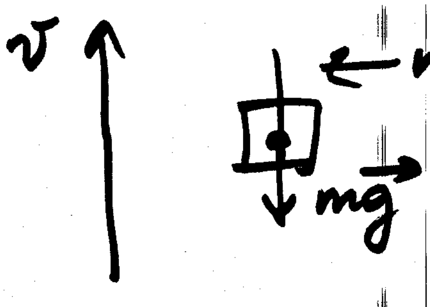
$$\vec{F} = m\vec{g} + \vec{F_1}$$

if body is going down, project on downward looking axis to get $F = mg - F_1$

if body is thrown up, $F = F_1 - mg$

From there, get a DE: $\boxed{\begin{array}{l} m\frac{dv}{dt} = mg - F_1 \\ \text{or} \\ m\frac{dv}{dt} = F_1 - mg \end{array}}$

Ex. Ball of mass 0.15 kg, thrown upward with initial velocity 20 m/sec from roof of building 30m high. Neglect air resistance.



← no air resistance

$F_1 = 0$.

$$\cancel{m}\frac{dv}{dt} = -\cancel{m}g$$

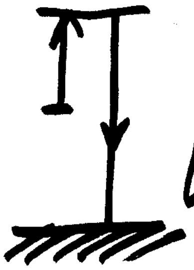
$$v' = -g \Rightarrow v = -gt + C$$

$$\boxed{v = v_0 - gt}$$

If $v = v_0 - gt$ and $x'(t) = v(t)$

$$\Rightarrow x = \int v(t)dt = \int (v_0 - gt)\,dt = \boxed{v_0 t - \frac{gt^2}{2} + C}$$

Max height is reached when $v(T) = 0$.



$$v(T) = V_0 - gT = 0$$

$$T = \frac{V_0}{g} = \frac{20}{g}\,[\text{sec}]$$

When it goes down,

$$m\frac{dv}{dt} = mg \Rightarrow v(t) = v_0' + gt = gt$$

$$x(t) = g\frac{t^2}{2} + C$$

Ball hits ground when

$$X(T_1) = g\frac{T_1^2}{2} + C = \underset{\text{building height}}{H} + \underset{\text{max height it reached in prev. step.}}{X_{max}}$$

If $F_1 \neq 0$ (air resistance) : $\frac{dv}{dt} = mg - F_1$

ex. $F_1 = \gamma|v|$ $\Rightarrow$

$$\frac{dv}{dt} = mg - \gamma v$$

$$\frac{dv}{dt} \overset{m}{=} \left(\frac{mg}{\gamma} - v\right)\gamma$$

$$\frac{dv}{\frac{mg}{\gamma} - v} = \gamma\,dt$$

$$\int \frac{dv}{v - \frac{mg}{\gamma}} = \int -\gamma\,dt$$

§2.4 Nonlinear vs. linear equations.

Thm 1. (Linear problem) $y'+p(t)y=g(t)$, $y(t_0)=y_0$ ⊛ IVP

If $p(t), g(t)$ are continuous in an interval $I: \alpha<t<\beta$ contains $t=t_0$, then there is a unique solution to ⊛ for any t in I and any y_0.

Pf. $y(t)=\frac{1}{\mu(t)}\left[\int_{t_0}^{t}\mu(s)g(s)ds+y_0\right]$, $\mu=e^{\int p(t)dt}$

integrable, cts functions.

Thm 2. Nonlinear problem $y'=f(t,y)$, $y(t_0)=y_0$ $(**)$ IVP

If $f(t,y)$ and $\frac{\partial f}{\partial y}(t,y)$ are continuous in $\alpha<t<\beta$, $\gamma<y<\delta$ containing (t_0,y_0), then there is a unique solution to $(**)$ in some interval $t_0-h<t<t_0+h$, contained in $\alpha<t<\beta$.

Remarks:

1) Thm2 reduces to Thm 1 if $f=-py+g$
2) If f is continuous in Thm 2, it is enough to guarantee existence of solution.
3) Graphs of 2 solutions of ⊛ or $(**)$ cannot intersect if the assumptions are satisfied.