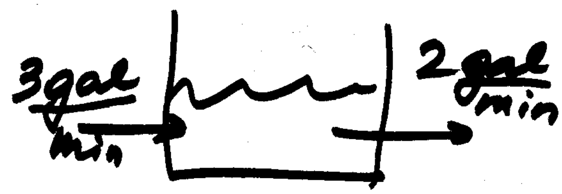
Math 214.002
Lecture 6.

Mixing problem.

brine solution in tank,
rate in = 3 gal/min
rate out = 2 gal/min, 300 gal initially
50 lbs of salt initially
inflowing concentration = 2 lb/gal



$A(t)$ = amount of salt at time t (lbs)
$Q(t)$ = total amount of solution at time t (gals).

1) $Q(t) = 300 + t$ (gal)
$Q(0) = 300$ (gal)

2) Wanted: equation for $\frac{dA}{dt}$

$$\frac{dA}{dt} = r_{in} \cdot C_{in} - r_{out} \cdot C_{out}$$
$$= 3\left[\frac{gal}{min}\right] \cdot 2\left[\frac{lb}{gal}\right] - 2\left[\frac{gal}{min}\right] \cdot \frac{A}{300+t}\left[\frac{lb}{gal}\right]$$

$$C_{out} = \frac{A(t)}{Q(t)} = \frac{\text{amount of salt}}{\text{total amount of solution}}$$
$$= \frac{A(t)}{300+t}$$

$$\boxed{A' = 6 - \frac{2A}{300+t}} \quad \underline{A(0) = 50} \leftarrow \text{IVP}$$

3) Solve this equation:

$$A' + \left[\frac{2}{300+t}\right] A = 6 \leftarrow \text{standard form for integrating factor}$$

$$p = \frac{2}{300+t}$$

$$\mu(t) = e^{\int p(t)dt} = e^{\int \frac{2}{300+t}dt} = e^{2\ln|t+300|}$$

$$= \left(e^{\ln(t+300)}\right)^2 = (t+300)^2$$

$(\mu A)' = 6\mu$ (from $A' + \frac{2}{300+t}A = 6$) multiplying both sides by $\mu(t)$

$$\mu A = \int 6\mu(t)dt + C$$

$$(t+300)^2 A(t) = 6\int (t+300)^2 dt + C$$

$$(t+300)^2 A(t) = 6 \cdot \frac{(t+300)^3}{3} + C \qquad \begin{matrix} u = t+300 \\ du = dt \end{matrix}$$

$$\boxed{A(t) = 2(t+300) + C(t+300)^{-2}}$$

$$A' + \frac{2}{300+t}A = 6$$

$$(t+300)^2 A' + 2(300+t)A = 6(300+t)^2$$

$$(\underbrace{(t+300)^2}_{\mu} A)'$$

$t=0 \quad A(0) = 50$

$$A(0) = 600 + \frac{C}{300^2} = 50$$

$$\lim_{t\to\infty} A(t) = \infty$$

time of overflow T: $\quad Q(T) = 500$
(limited capacity say 500 gal)

$$Q(T) = 300 + T = 500$$

$$T = 200$$

<u>Now:</u> Let $r_{in} = 2$ gal/min
$r_{out} = 3$ gal/min

Q1. Find amount of solution at time t

Q2. Write down a DE for the amount of Salt at time t.

Q3. Solve this DE to get $A(t)$.