Answers to Spring 2003 Final exam.

Q#:1 Answer D.
Q#:2 Answer C.
Q#:3 Answer C.
Q#:4 Answer B.
Q#:5 Answer B.
Q#:6 Answer A.
Q#:7 Answer A.
Q#:8 Answer C.
Q#:9
A Solution exists on \((0, \infty)\).
B The solution is \(y(t) = c_1 t + c_2 \frac{1}{t}\).
Q#:10
A The linearized matrix is 
\[
\begin{bmatrix}
y - 3 & x \\
2y & 2x + 2
\end{bmatrix}
\].
B The critical points are \(P = (0, 0)\) and \(Q = (-1, 3)\).
C Point \(P\) is a saddle point, which is unstable. Point \(Q\) is a centre, which is stable.
Q#:11

A The linearized matrix is 
\[
\begin{bmatrix}
y - 3 & x \\
2y & 2x + 2
\end{bmatrix}
\].
B The critical points are \(P = (0, 0)\) and \(Q = (-1, 3)\).
C Part (a) has a cosine series and part (b) has a sine series.
D \(f(-2) = 0, f(\frac{1}{2}) = \frac{1}{2}\) and \(f(3) = -\frac{3}{2}\).
Q#:12 Denoting \(u(x, y) = F(x)G(y)\),
A \(G''(y) + 2G'(y) - \lambda y^2 G(y) = 0\) and \(F''(x) - \lambda F(x) = 0\) are the two ordinary differential equations.
B The boundary conditions become \(F(0) = F(L) = 0\).
Q#:13
\[
\lambda_n = \frac{(2n - 1)^2}{4}, \quad f_n(t) = \sin \left(\frac{(2n - 1)t}{2}\right), \quad n \in \mathbb{N}
\]
Q#:14
A \(u_t = 2u_{xx}\) \(u_x(0, t) = u_x(10, t) = 0\) \(u(x, 0) = 3t \cos \left(\frac{\pi x}{5}\right) - 5 \cos \left(\frac{\pi x}{5}\right)\).
B \(u(x, t) = 3 + \exp \left(\frac{-2\pi^2 t}{25}\right) \cos \left(\frac{\pi x}{5}\right)\).
C The steady state emperature is 3.
Q#:15
A The general solution is \(X(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \left( \begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-t} \right)\).
B The specific solution is \(X(t) = -\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + 3 \left( \begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-t} \right)\).