Random Variables

Definition. A random variable is a function

$$X : \Omega \to \mathbb{R}.$$

Discrete random variable: has countable set of values (finite or countably infinite).

Has an associated probability mass function

Example: Roll pair of dice; X is the sum of the numbers that come up.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The PMF:

k	2	3	4	5	6	7	8	9	10	11	12
$P(X=k)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Random Variables

X

$X : \Omega \to \mathbb{R}$

X gets a value when the experiment is done. Before that, we can talk about $P(X = x)$, where $x \in \mathbb{R}$, or $P(a < X < b)$, etc.

Bernoulli Random Variables

X can have only the values 0 and 1.

k	0	1
$m_X(k)$	p	$q = 1-p$

Expected Value of a Random Variable (discrete case)

$$E(X) = \sum_{\omega \in \Omega} \omega P(X = \omega).$$

Examples.

① A coin is flipped. X is the number of Heads (0 or 1).

$$E(X) = 0 \cdot \tfrac{1}{2} + 1 \cdot \tfrac{1}{2} = \tfrac{1}{2}.$$

② A die is rolled. Y is the number that comes up.

$$E(Y) = 1 \cdot \tfrac{1}{6} + 2 \cdot \tfrac{1}{6} + 3 \cdot \tfrac{1}{6} + 4 \cdot \tfrac{1}{6} + 5 \cdot \tfrac{1}{6} + 6 \cdot \tfrac{1}{6}$$

$$= \binom{7}{2} / 6 = \tfrac{7}{2}.$$

Homework

3.1, 3.2, 3.3, 3.8 - 12, 3.16, 3.29.