Axioms

The sample space, Ω.

Events, $E \subseteq \Omega$.

Axioms of the probability function, P:

1. $P(A) \geq 0$ for each event A;
2. $P(\Omega) = 1$;
3. If $A_1, A_2, \ldots$ is a sequence of events and these events are pairwise disjoint, then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

Some Rules Implied by the Axioms

① $P(A_1 \cup \dots \cup A_n) = P(A_1) + \dots + P(A_n)$,
if the events $A_1, \dots, A_n$ are mutually exclusive (that is, pairwise disjoint).

② $P(A) = 1 - P(A^c)$.

③ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

④ $$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{1 \le i < j \le n} P(A_i \cap A_j)$$

$$+ \sum_{1 \le i < j < k \le n} P(A_i \cap A_j \cap A_k) - \dots + (-1)^{n-1} P(A_1 \cap \dots \cap A_n).$$

Compound Chance Experiments

n physically independent experiments.

$\Omega = \Omega_1 \times \Omega_2 \times \dots \times \Omega_n$ ← sample space,

$$P(E_1 \times E_2 \times \dots \times E_n) = P(E_1)P(E_2)\dots P(E_n).$$

Examples;

Tossing a die twice; or tossing a pair of dice,

Tossing a die, then flipping a coin.

Flipping a coin n times,

Birthday Problem

A class has n students. What is the probability that some two have the same birthday?

Hat Problem

Suppose n people check their hats at the door when entering, but when leaving their hats are returned at random. What is the probability that no one receives the right hat?

Homework

Suggestions for problems to work —

1.38 – 1.43, 1.44, 1.45, 1.47, 1.48, 1.49,
1.51, 1.54, 1.58, 1.78, 1.84.