Test, next Wednesday!

Ch. 0, Ch. 1, Ch. 2.

Monday - Review.

For n - an integer, $n \geq 2$.

"$a \equiv b$ modulo n"

congruent

means

$n \mid (b-a)$

(There is an integer m such that $b - a = m \cdot n$.

When $n = 5$ –

the numbers $\ldots, -10, -5, 0, 5, 10, \ldots$

(the multiples of 5) are all congruent modulo 5.

The same is true for:

$\ldots -9, -4, 1, 6, 11, \ldots$;

$\ldots -8, -3, 2, 7, 12, \ldots$;

$\ldots -7, -2, 3, 8, 13, \ldots$;

$\ldots -6, -1, 4, 9, 14, \ldots$.

Proof that $a \equiv b \bmod. n$ is an equivalence relation?

Suppose $z \in \mathbb{Z}$. Then $z - z = 0 \cdot n$, so $z \equiv z \bmod. n$.
So the relation is reflexive.

Suppose $w \equiv z \bmod. n$. Then there is an integer $m \in \mathbb{Z}$ for which $z - w = m \cdot n$. Then $w - z = (-m) \cdot n$, so, since $-m \in \mathbb{Z}$, $z \equiv w \bmod n$.
So the relation is symmetric.

Suppose $x \equiv y \bmod n$ and $y \equiv z \bmod n$. Then there are $m_1, m_2 \in \mathbb{Z}$ for which $y - x = m_1 n$ and $z - y = m_2 \cdot n$.

Then $z - x = (z - y) + (y - x)$
$= m_2 \cdot n + m_1 \cdot n = (m_2 + m_1) \cdot n$.
Since $m_1 + m_2 \in \mathbb{Z}$, $x \equiv z \bmod n$.
The relation is transitive. $\square$

S - a set

ρ - an equiv. relation on S

$s\rho t$ means $(s,t) \in \rho$

If $s \in S$ then the equivalence class containing s is

$$\bar{s} = \{ t \in S \mid s\rho t \}.$$

A partition of S is a collection of sets $\{S_1, \ldots, S_n\}$ such that

① For each i, $S_i \neq \emptyset$,

② If $i \neq j$, $S_i \cap S_j = \emptyset$,

and ③ $S = S_1 \cup S_2 \cup \ldots \cup S_n$.

Suppose $\{S_1, S_2, \ldots, S_n\}$ is a partition of S. Define the relation ρ on S by "$s \rho t$" means there is i for which $s \in S_i$ and $t \in S_i$.

Then ρ is an equivalence relation on S.

THEOREM. Any partition of a set S determines an equivalence relation on S, as above. Conversely, if ρ is an equivalence relation on a set S, then the equivalence classes of ρ form a partition of S.

Partial Ordering Relations

$\rho \subseteq A \times A$ is a partial ordering relation on A if it is reflexive, antisymmetric, and transitive.

Examples: $\leq$, on $\mathbb{R}$; $\subseteq$, on $\mathcal{P}(S)$; "divides" on $\mathbb{N}$.

$\mathcal{P}(\{1, 2, 3\})$

$A \subseteq B$ and $B \subseteq A \Rightarrow A = B$

(antisymm.)

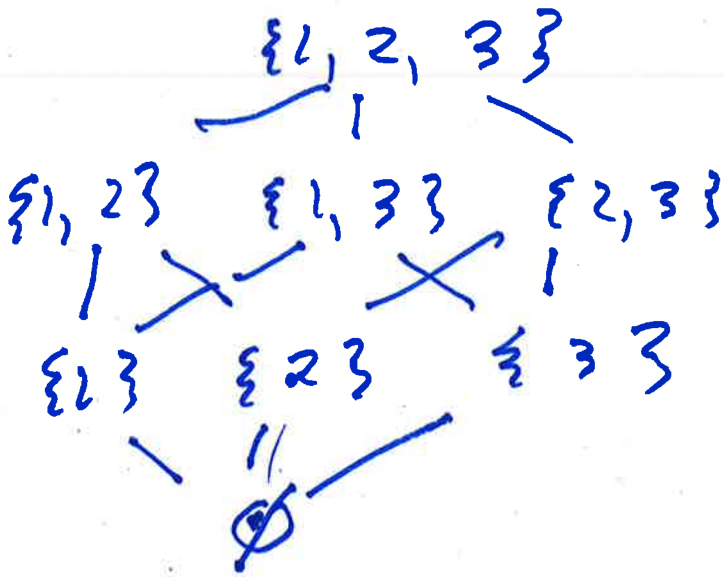
$A \subseteq A$ (refl.)

$A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$

trans.

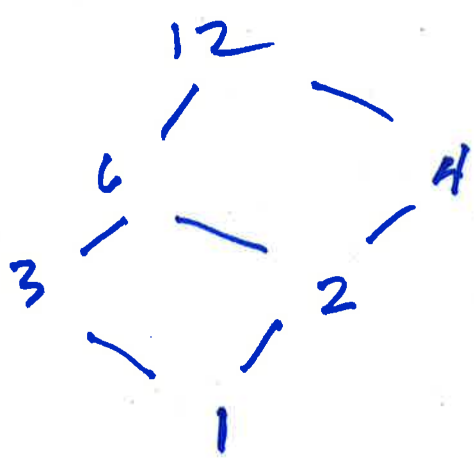
$\subseteq$ partial ordering relation.

Hasse Diagrams



$\subseteq$ on $\mathcal{P}(\{1,2,3\})$

Divisors of 12 (in $\mathbb{N}$)



Homework

2.5: T-F and 1-5, 7, 9(a).