

If  $x$  is a prime number  
then  $x$  is not a square number.

Proof. Suppose  $n$  is a square number. Since  $n$  is a square number, there is a positive integer  $m$  such that  $n = m^2$ . Then  $m$  is a divisor of  $n$ . If  $m = 1$  then  $n = 1$ , so  $n$  isn't prime. If  $m > 1$  then  $1 < m < m^2 = n$ , so  $n$  is not a prime number.  $\square$

Definition. If  $n$  is a positive ~~number~~ integer and  $d$  is a positive integer, then  $d$  divides  $n$  if there is an integer  $k$  such that  ~~$n = k \cdot d$~~   $n = k \cdot d$ .

If  $x$  is a prime number  
then  $x$  is not a square  
number.

Proof. Suppose  $x$  is a  
prime number. Then  $x \neq 1$ .  
If  $1 \leq k < x$  then  $x \neq k^2$  since  
otherwise  $k$  would be a divisor of  $x$   
that is not 1 or  $x$ .

So  $x$  is not a square number.  $\square$

P  
If  $x$  is odd and  $y$  is odd then  $x+y$  is even.  $Q$

(an implication.)

$$P \rightarrow Q$$

What if  $Q$  is false?

$$\neg Q \rightarrow \neg P \leftarrow \text{contrapositive of } P \rightarrow Q$$

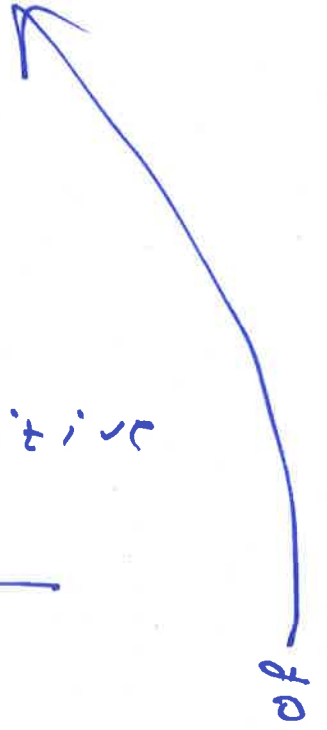
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$$\neg Q \rightarrow \neg P$$

~~If  $x+y$  is not even~~

If  $x+y$  is odd then  
 $x$  is even or  $y$  is even.

contrapositive  
negation



The ball is red and the  
marble is green.

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The ball is not red  
or the marble is not green.

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## Quantifiers

For each

There exists

Every marble in the urn  
is green.

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There is a marble in the  
urn that's not green.

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There is a dog that  
doesn't bark.

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All dogs bark.

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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For every pair of integers  $x$  and  $y$ , if  $x$  and  $y$  are both odd then  $x+y$  is even.

"For every"

"there exists"