p: Rover barks.

q: Jane has brown hair.

r: George is 6'5" tall.

simple statements.

(not compound statements)

Compound statements:

Rover barks and Jane has brown hair.

Logical symbols

If p, q are statements, then –

$p \wedge q$ means "p and q"

$p \vee q$ means "p or q"

$\neg p$ means "not p"

$p \rightarrow q$ means "if p then q"

$p \leftrightarrow q$ means "p if and only if q"

Tautology

Contradiction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p and q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p or q

p	$\neg p$
T	F
F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

"$p \leftrightarrow q$" means the same thing as "$(p \wedge q) \vee (\neg p \wedge \neg q)$."

---

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

"if p then q."

Also – means

$\neg(p \wedge \neg q)$

$$p \leftrightarrow q \underset{\leftrightarrow}{\Longleftrightarrow} (p \rightarrow q) \wedge (q \rightarrow p).$$

"p if and only if q"

p implies q and q implies p.

A statement of the form $p \to q$ is called an <u>implication</u>.

When a statement is an implication, it has a converse and a contrapositive, which are also implications.

statement: $p \to q$

converse of : $q \to p$

contrapositive of $\neg q \to \neg p$.

$p \to q$ implication

$q \to p$ converse

$\neg q \to \neg p$ contrapositive

---

p	q	$p \to q$	$q \to p$	$\neg q \to \neg p$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

the same

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$p \leftrightarrow q$

If $\overbrace{x \text{ and } y \text{ are odd}}^{p}$ then $\underbrace{x+y \text{ is even}}_{q}$.

$p \to q$. $\neg q \to \neg p$.

p: "the marble is red"

q: "it is raining"

$p \to q$ ← an "implication"

hypothesis   conclusion

$\neg(p \wedge q) \vee (p \wedge q)$

always true

tautology

Some Identities

See the list of identities on page 24 of the text.

De Morgan's Laws:

$$\neg(p \vee q) \Leftrightarrow (\neg p) \wedge (\neg q)$$

$$\neg(p \wedge q) \Leftrightarrow (\neg p) \vee (\neg q)$$

Distributive Laws:

$$p \wedge (q \vee r) \Longleftrightarrow (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \Longleftrightarrow (p \vee q) \wedge (p \vee r).$$

Disjunctive Normal Form

$(a+b)\cdot c$ (distributive law)

$= a\cdot c + b\cdot c$

$\underline{a+(b\cdot c)} \overset{?}{=} (a+b)\cdot(a+c)$

$2+3\cdot 4 = 14$

$\neq 20$

$$\boxed{\begin{aligned}&(2+3)\cdot 4\\ &= 2\cdot 4 + 3\cdot 4.\end{aligned}}$$

$\not\leftarrow$ $\neg(p \wedge \neg q)$

$(\neg p)\wedge(\neg q)$

$2+(3\times 4)$

$\neq$

$(2+3)\times(2+4)$

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

p	q	r	
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$$

$$p \wedge \neg(q \wedge r) \Leftrightarrow p \wedge (\neg q \vee \neg r)$$

$$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r)$$

p	q	r	
T	F	T	T
T	F	F	T
T	T	F	T

$\neg(p \vee q)$

equiv.

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

The negation of the statement "Rover barks or Jane has brown hair" is

Rover doesn't bark AND Jane doesn't have brown hair.

De Morgan's Laws

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$

$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \vee (q \wedge r) \Leftrightarrow$

p	q	r	
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

"Disjunctive normal form"

$(p \wedge \neg q) \vee (\neg p \wedge q)$

p	q	$(\ )$
T	T	F
T	F	T
F	T	T
F	F	F

$(p \wedge q) \vee (p \wedge \neg q \wedge \neg r)$

p	q	r	
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$

TRUE or False

1. If x is odd and y is odd then $x+y$ is even. | True.

For every integer ~~and~~ x and every integer y, if x is odd and y is odd then $x+y$ is even.

$\forall x$ for every x

$\exists x$ there exists x such that

Homework

The true-false problems at the end of each section

1.1: 1, 2, 3, 4, 5, 10;

1.2: 3, 6.