Mid-Atlantic Algebra Conference

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George Mason University, Fairfax, Virginia

Abstracts

Invited Talks:

Algebraic Statistics, I and II

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Abstract

Algebraic statistics is concerned with the study of probabilistic models and their applications to data analysis from the point of view of (computational) algebraic geometry. The aim of these lectures is to offer algebraists an introduction to this subject. Special emphasis is placed on models used in biology and on the problem of maximum likelihood estimation.

Contributed Talks:

Characters and Solvable Groups

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Abstract

Let G be a finite group. Let χ and ψ be irreducible complex characters, i.e. irreducible characters over the complex numbers. Since a product of characters is a character, $\chi\psi$ is a character. Then the decomposition of the character $\chi\psi$ into its distinct irreducible constituents $\alpha_1,\ \alpha_2,\ldots,\alpha_n$ has the form

$$\chi\psi = \sum_{i=1}^{n} a_i\alpha_i$$

where $n > 0$ and $a_i > 0$ is the multiplicity of α_i for each $i = 1,\ldots,n$. Let $\eta(\chi\psi) = n$ be the number of distinct irreducible constituents of the character $\chi\psi$. Denote by $\mathrm{Ker}(\chi\psi)$ the kernel of the character $\chi\psi$. Also denote by $\mathrm{dl}(\mathrm{Ker}(\alpha)/\mathrm{Ker}(\chi\psi))$ the derived length of the group $\mathrm{Ker}(\alpha)/\mathrm{Ker}(\chi\psi)$. The main result of this work is about the existence of universal constants C and D such that for any finite solvable group G, any irreducible characters χ and ψ of G and any irreducible constituent α of $\chi\psi$, we have $\mathrm{dl}(\mathrm{Ker}(\alpha)/\mathrm{Ker}(\chi\psi)) \leq C\eta(\chi\psi) + D$. If the group G is, in addition, supersolvable then we may take $C = 2$ and $D = -1$.

Also we proved that if $(\chi(1),\psi(1)) = 1$, then the irreducible constituents of the product $\chi\psi$ are "almost" faithful characters of the group $G/\mathrm{Ker}(\chi\psi)$.

Computational Algebraic and Game Theoretic Techniques in Modelling Protein Signalling Networks

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Abstract

A key issue in the study of protein signaling networks is understanding the relationships between proteins in the network. Understanding these relationships in the context of a network is one of the major challenges for modern biology (E. Alm and A. P. Arkin; A. Levchenko).

Laubenbacher and Stigler have developed an algorithm that uses Gröbner bases to analyze time-series data generated by a cell. Essentially, their algorithm constructs Lagrange interpolation polynomials that represent next-state functions for the data. By analyzing the relations of the proteins as variables in polynomials over a quotient ring with coefficients from a finite field $\mathbb{Z}_p$, they construct conjectures about mathematical models of the cell. This idea generalizes the analysis of networks since $\mathbb{Z}_2[x_1, \ldots, x_n]$ is a Boolean algebra. One important aspect of Buchberger's algorithm in the creation of Gröbner bases is variable order. Essentially, Buchberger's algorithm provides a solution that is a representative of a class of solutions. The variable order gives Buchberger's algorithm information on which solution to choose.

Laubenbacher and Stigler left two open questions with respect to their work: (1) what is the optimal variable order; and (2) what is the optimal way to discretize the time-series data. This presentation focuses on an application of some Game Theoretic statistics to conjecture dependencies between proteins in signal transduction networks. These statistics give conjectures concerning the optimal variable order. (This is joint work with Jacquelyn S. Fetrow, David J. John and Stan J. Thomas.)

Half-Factorial Domains

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Abstract

An integral domain R is a half-factorial domain (HFD) if each nonzero, nonunit of R is a product of irreducible elements and any two factorizations of a given nonzero, nonunit of R have the same number of irreducible factors. For example: any UFD is an HFD, but $\mathbb{Z}[\sqrt{-5}]$ is an HFD, but not a UFD. We will discuss an invariant (due to Scott Chapman) which measures, in some sense, how far an HFD is from being a UFD.

Representations of the Lie Pseudoalgebra $W(\mathfrak{d})$

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Abstract

One of the algebraic structures that has emerged recently in the study of the operator product expansions of chiral fields in conformal field theory is that of a Lie conformal algebra. A Lie pseudoalgebra is a generalization of the notion of a Lie conformal algebra for which $\mathbb{C}[\partial]$ is replaced by the universal enveloping algebra $U(\mathfrak{d})$ of a finite-dimensional Lie algebra $\mathfrak{d}$. One can construct a simple Lie pseudoalgebra $W(\mathfrak{d})$, which is closely related to the Lie–Cartan algebra W_N of vector fields, where $N = \dim \mathfrak{d}$. Our main result is the classification of all irreducible finite $W(\mathfrak{d})$-modules. (Based on a joint work with A. D'Andrea and V. G. Kac.)

Renormalization, Bonsai and Homological Algebra

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Abstract

As an approximation to the Hopf algebra introduced by Connes-Kreimer, we construct a new Hopf algebra structure by considering actual shapes of Feynman diagrams. It has a basis consisting of forests of tree diagrams having a finite upper bound of their branching numbers. We call such a tree diagram "bonsai", show that it involves an operad structure, derive from that some cochain complex structures which are quite natural from the viewpoint of operad theory, and present some results on their homological algebra.

Affine Lie Algebras and Multisum Identities

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Abstract

Affine Lie algebra representatations have many connections with different areas of mathematatics and physics. One such connection in mathematics is with number theory in general and combinatorial identities in particular. In this talk we consider level k (k a positive integer) integrable highest weight representations of affine Lie algebras of ADE type. Viewing these representations as vertex operator algebras and using vertex operator algebra methods we obtain recurrence relations for their characters. We then solve these recurrence relations for the case where $k = 1$. Taking the pricipal specialization we obtain new families of multisum identities of Rogers-Ramanujan type.

Computation of Noetherian Operators

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Abstract

In this talk I present some algorithmic techniques to compute explicitly the noetherian operators associated to a large class of ideals and modules over a polynomial ring. The procedures I describe can be easily encoded in computer algebra packages such as CoCoA and Singular. I will show some examples both for zero-dimensional ideals and modules as well as some non zerodimensional ideals.

Differential Calculus on Quantum Homogeneous Spaces

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Abstract

The theory of quantum groups provides numerous examples of noncommutative analogs of coordinate rings of affine algebraic varieties with group action. S. L. Woronowicz posed the question if these examples admit an analogue of Kähler differentials. He called a derivation of an algebra into a bimodule which is compatible with a quantum group action a "covariant first order differential calculus" (S. L. Woronowicz). The existence of a covariant first order differential calculus can also be considered as an obstruction to establishing these algebras as examples of noncommutative geometry in the sense of A. Connes.

In this talk the theory of covariant first order differential calculi over a large class of quantum homogeneous spaces considered by E. F. Müller and H.-J. Schneider is reviewed. It turns out that for these algebras the classification of covariant differential calculi is equivalent to a purely representation theoretic problem. An example for which classification has been obtained (I. Heckenberger and S. Kolb) are quantized irreducible flag manifolds, which have recently been linked to noncommutative geometry by U. Krähmer.

Symmetric Brace Algebras

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Abstract

Symmetric brace algebras are a natural generalization of the more familiar brace algebras. Just as brace algebras have been used to study operations in the Hochschild complex of an associative algebra, symmetric braces may be used to study operations in the Chevalley-Eilenberg complex of a Lie algebra. In particular, brace algebras describe A-infinity algebra structures while symmetric braces describe L-infinity algebras.

The Maximal Subalgebras of the General Lie Algebra

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Abstract

The description of maximal subsystems of any algebraic system is an essential and important step toward the structural characterization of the system. In the Lie Theory there are a number of well-known problems where the maximal subsystems play crucial role.

In this talk we will discuss results concerning maximal subalgebras of the General Lie algebras of Cartan W_n over an algebraically closed field of characteristic zero. In particular, the complete classification of graded maximal subalgebras of W_n is obtained. It also will be shown that there are only finitely many conjugacy classes of maximal graded subalgebras in W_n under the group of automorphisms of W_n. Moreover, the representatives of the conjugacy classes of maximal subalgebras will be explicitly constructed.

Alternative Theorems and the Duality Theorems of Linear Programming

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Abstract

For solving the linear programming problems (LP's), we now have two main classes of algorithms. One is the class of simplex methods, which was first developed by G. Dantzig. The other is the class of interior point methods. Although both are very sophisticated and very efficient for solving, the finiteness of their algorithms and the implementations are still hard for the beginner. Even in many published LP books, proofs of the duality theorems make use of the accuracy of the algorithms.

With this in mind, we give a fresh look at the alternative theorems. We will show an extremely simple constructive proof of alternative theorems, which directly yields a recursive algorithm for solving LP's. Our notion will be extended to the setting of conic systems.

A Category of Discrete Posets

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Abstract

In several books on category theory, the category "Poset" consisting of all partially ordered sets (posets) together with all order preserving functions as the set of morphisms is presented as an introductory example. It is appealing because posets are a well known structure and it is evident that the composition of order preserving functions is order preserving. But, unfortunately it is not a "category"! Order preserving functions need not take posets into posets.

In our talk we present a valid poset category based on closure operators.

A New Approach to Hilbert's Theorem on Ternary Quartics

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Abstract

Hilbert proved that a non-negative real quartic form $f(x, y, z)$ is the sum of three squares of quadratic forms. We give a new proof which shows that if the plane curve Q definte by f is smooth, then f has exactly 8 such representations, up to equivalence. They correspond to those real 2-torsion points of the Jacobian of Q which are not represented by a conjugate-invariant divisor on Q. This is joint work with Bruce Reznick, Claus Scheiderer, and Frank Sottile.

An Algebraic Approach to Modeling Dynamic Biological Systems

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Abstract

One of the central problems in molecular biology is to understand the dynamic network of interactions among genes and proteins in a regulatory network. One approach to this problem is to construct mathematical models of such networks, based on experimental data, commonly referred to as reverse engineering or top-down modeling. Traditionally continuous models, described by systems of differential equations, have been used. Of increasing interest in mathematical modeling is the use of discrete models, given by polynomials defined over finite fields. In this discrete setting, computational algebraic geometry can be exploited for its powerful machinery in solving polynomial systems.

In this talk I will introduce a discrete modeling method, which uses algorithmic tools from algebraic geometry, to build all possible polynomial models that fit a given time series of data. A technique to select biologically relevant models will follow. I will conclude with a discussion of the appropriateness of data for the modeling method.

Counting Conics on the Cubic Surface

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Abstract

A nice problem in classical algebraic geometry is to count the number of conic curves tangent to five given conics in the projective plane. I'll review a solution to this problem using intersection theory and then turn to the related problem of counting conic curves that lie on the cubic surface and satisfy certain geometric constraints. This is joint work with my colleagues Amy Ksir and MIDN Andrew Bashelor.

Braid Group Actions on Semisimple Tensor Categories

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Abstract

Semisimple tensor categories arise in knot theory, the representation theory of quantum groups, Kac-Moody algebras, and loop groups. They are essential components of conformal field theory and quantum field theories as well. In such quantum mechanical systems, certain "symmetries" arise in the form of a braiding. Under suitably nice conditions, the braiding gives a representation of the braid group into a series of semisimple algebras.

I will talk about how this happens and how understanding such braid representations helps in characterizing the entire tensor category. The question is mostly open and is the subject of ongoing research, but Kazhdan, Wenzl, and I have complete results in certain special cases related to representation categories of classical type quantum groups. These are based on a comparison of the Markov trace and the categorical trace.

Nilpotent n-Lie Algebras

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Abstract

In 1986, Kasymov introduced the concept of nilpotent n-Lie algebras, proved an analogue of Engel's Theorem and later proved an analog of Jacobson's refinement of Engel's Theorem. Despite these achievements, the subject of nilpotency in n-Lie algebras has not been examined in great detail in the literature since. We shall explore the concept of nilpotent n-Lie algebras by examining, and proving where possible, other classical nilpotent group theory and nilpotent Lie algebra results, in the n-Lie algebra setting.