(1) (3.26) Prove that if $(ab)^2 = a^2b^2$, for $a, b \in G$, then $ba = ab$.

Proof: We have $(ab)^2 = abab = a^2b^2 = aabb$. We can cancel on the left by a (or if you prefer, multiply both sides on the left by a^{-1}) and cancel on the right by b. Then we obtain $ba = ab$.

(2) (3.18) If a and b are distinct elements of the group G, prove that either $a^2 \neq b^2$ or $a^3 \neq b^3$

Proof: We will do a proof by contradiction. Suppose that both equalities hold. Then $a^3 = b^3 = b(b^2) = b(a^2)$. Now cancel an a^2 from both sides of the equation $a(a^2) = a^3 = b(a^2)$ and we get $a = b$ a contradiction.

(3) (3.34) Find the six cyclic subgroups of $U(15)$.

Proof: $U(15) = \{1, 2, 4, 7, 8, 11, 13, 14\}$. We will examine the cyclic subgroup generated by each element of G.
$\langle 1 \rangle = \{1\}$
$\langle 2 \rangle = \{2, 4, 8, 1\}$
$\langle 4 \rangle = \{4, 1\}$
$\langle 7 \rangle = \{7, 4, 13, 1\}$
$\langle 8 \rangle = \{8, 4, 2, 1\} = \langle 2 \rangle$
$\langle 11 \rangle = \{11, 1\}$
$\langle 13 \rangle = \{13, 13^2 = 169 = 4, 7, 1\} = \langle 7 \rangle$
$\langle 14 \rangle = \{14, 1\}$
Thus there are six distinct cyclic subgroups; the subgroups generated by $1, 2, 4, 7, 11, 14$.

(4) (3.48) Let $H = \{a + bi | a, b \in \mathbb{R}, ab \geq 0\}$. Is H a subgroup of $\mathbb{C}$ under addition?

Proof: We will show that H is not a subgroup by showing that there exists elements $x, y \in H$ such that $x + y \notin H$. Let $x = 3 + i$ and $y = -2 - 2i$. Then x and y are elements of H. But $x + y = 1 - i$, which is clearly not an element of H. Thus H is not a subgroup.