

NAME (print): KEY

Math 108 Summer 2009—Exam 3

Instructor: J. Shapiro

Work carefully and neatly and remember that I cannot grade what I cannot read. You must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Notes, books, and graphing calculators are NOT ALLOWED. Please circle your answer where appropriate.

[10pt]

1. A manufacturer's total monthly revenue is  $R(x) = 240x + 0.05x^2$  dollars when  $x$  units are produced during the month. Currently, the manufacturer is producing 80 units a month and is planning to decrease the monthly output by .7 units. Use calculus to estimate how the total monthly revenue will change as a result.

$$\Delta R \approx R'(x_0) \Delta x$$

$$x_0 = 80$$

$$\Delta x = -.7$$

$$R'(x) = 240 + .1x$$

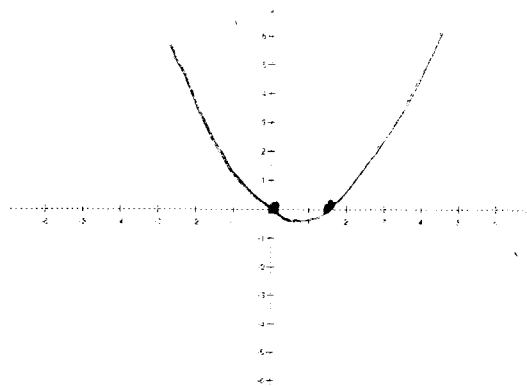
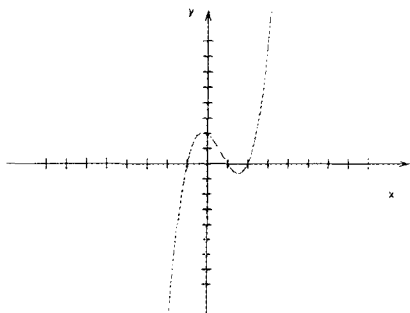
$$R'(80) = 240 + (.1)(80) = 248$$

$$R'(x_0) \Delta x = 248(-.7)$$

$$= -173.6 \text{ dollars}$$

[16pt]

2. Let the the function  $f$  given by the graph below, left.



- (a) Find the intervals where  $f$  is increasing and where it is decreasing.

$$f \text{ increasing on } (-\infty, 0) \cup (3/2, \infty)$$

$$f \text{ decreasing on } (0, 3/2)$$

- (b) Sketch a possible graph for  $f'$  on the coordinate axis above right.

- [10] 3. For the function  $f(x) = 4x^3 - 12x^2 + 9x$  find all critical numbers and then use the second derivative test to determine which (if any) give a relative max or a relative min. If the second derivative test fails, then you should use the first derivative test.

$$f'(x) = 12x^2 - 24x + 9 = 3(4x^2 - 8x + 3)$$

$$f' = 0 \Rightarrow 4x^2 - 8x + 3 = 0$$

$$(2x - 3)(2x - 1) = 0$$

$$x = 3/2 \quad x = 1/2$$

$$f'' = 24x - 24$$

$$f''(3/2) = 24(3/2) - 24 = +$$

$$f''(1/2) = -$$

$$\boxed{3/2 - \text{min}, \quad 1/2 - \text{max}}$$

- [10] 4. Sketch the graph of the following function  $f(x) = x^3 - 3x^2 + 2$ . Include all y-intercepts, local extrema, and points of inflection.

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0, \quad x = 2$$

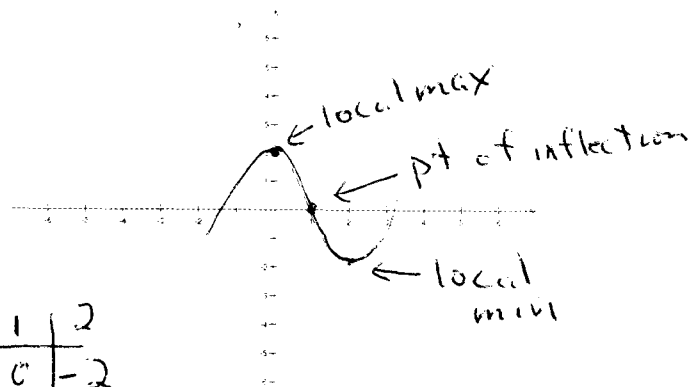
$$f''(x) = 6x - 6 = 0$$

$$x = 1$$

$$f''(0) = - \quad \text{max}$$

$$f''(2) = + \quad \text{min}$$

$$x = 1 \text{ pt of inflection}$$



x	0	1	2
y	2	0	-2

- [10pt] 5. Let  $f(x) = \frac{x+2}{(x-1)^2}$ . Then  $f'(x) = -\frac{x+5}{(x-1)^3}$ . Determine on what intervals  $f(x)$  is increasing and on what intervals it is decreasing.

$$f' = 0 \text{ or undefined}$$

$$x = -5, \quad x = 1$$

$$-6, \quad 0, \quad 2$$

x	f'
-6	-/- = +
0	+/- = -
2	+/- = +

Increasing:  
 $(0, -5)$  and  $(1, \infty)$

Decreasing  
 $(-5, 1)$

- [10pt] 6. Let  $f(x) = x^4 - 24x^2 + 10$ . Find the intervals where  $f$  is concave up and where it is concave down.

$$f' = 4x^3 - 48x$$

$$f'' = 12x^2 - 48 = 0$$

$$12(x^2 - 4)$$

$$x = -2, 2$$

$$-3, 0, 3$$

x	f''
-3	12(+) = +
0	12(-) = -
3	12(+) = +

Concave down  
 $(-2, 2)$

Concave up  
 $(-\infty, -2)$   
and  $(2, \infty)$

- [10] 7. Find the absolute maximum and minimum values of the function and where they occur for  $f(x) = -3x^4 + 8x^3 - 10$  on the interval  $-1 \leq x \leq 1$ .

$$f'(x) = -12x^3 + 24x^2 = 0? \quad \left| \quad \begin{array}{l} f(-1) = -21 \\ f(0) = -10 \\ f(1) = -5 \end{array} \right.$$

$$-12x^2(x-2) = 0?$$

$$x=0$$

$$x=2 \text{ - not on interval}$$

$$\begin{array}{l} \text{max} = -5 \text{ at } x=1 \\ \text{min} = -21 \text{ at } x=-1 \end{array}$$

- [10] 8. Give the horizontal and vertical asymptotes for the function  $f(x) = \frac{x^3 + x + 1}{x^3 - x}$ .

V.A.  $x^3 - x = 0$   
 $x(x^2 - 1) = 0$   
 $x=0, x=-1, x=1$

H.A.  $\lim_{x \rightarrow \infty} f(x) = 1$   
 $y=1$

- [14] 9. Sketch the graph of the following function. Be sure to include the following:
1. asymptotes.
  2. intercepts and
  3. relative max and mins.

$$f(x) = \frac{2x^2}{x^2 - 4}. \quad (\text{Note that } f'(x) = \frac{-16x}{(x^2 - 4)^2}).$$

V.A.  $x = -2, x = 2$

H.A.  $y = 2$

$x=0, y=0$

Intercept:  $(0,0)$

$f'(-1) = +$  /

$f'(1) = -$  \

~~0~~,  $f'(x) = 0$  so

$x=0$  - min

