NAME (print): ______________________________

Math 108 Summer 2009—Exam 2

Instructor: J. Shapiro

Work carefully and neatly and remember that I cannot grade what I cannot read. In the long answer questions you must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Place your answers in the boxes provided. Notes, books and graphing calculators are NOT ALLOWED.

[25pt] 1. Find the limit, if it exists for the following functions (otherwise select DNE). Circle the correct answer.

(a) $\lim\limits_{x \to 3} \dfrac{x^2 - 2}{x + 3}$

a) 1 b) 7/6 c) 0 d) 7 e) DNE

(b) $\lim\limits_{x \to 3} \dfrac{6x^2 - 4}{x - 3}$

a) 0 b) 50 c) 3/7 d) 5/3 e) DNE

(c) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 3x + 2}$

a) -1 b) 1 c) -3 d) 5 e) DNE

(d) $\lim\limits_{x \to \infty} \dfrac{5x - 5}{x^2 - 3x + 2}$

a) -5 b) 0 c) 2/3 d) $-1/5$ e) ∞

(e) $\lim\limits_{x \to \infty} \dfrac{6x^4 - 3x^2}{2x^4 - 5x - 2}$

a) 3 b) 5 c) 0 d) ∞ e) DNE

[9pt] 2. Using the limit definition of the derivative, show that if $f(x) = x^2 + 2x$, then $f'(x) = 2x + 2$.

Answer:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 + 2(x+h) - [x^2 + 2x]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$= \frac{2xh + h^2 + 2h}{h}$$

$$= \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$

$$\rightarrow 2x + 2$$

[8pt] 3. Find the equation of the tangent line to the curve $y = x + \sqrt{x}$ at the point where $x = 4$.

Answer: $y = \frac{5}{4}x + 1$

$y = x + x^{1/2}$

$y' = 1 + \frac{1}{2}x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}$

$y'(4) = 1 + \frac{1}{2\sqrt{4}} = 1 + \frac{1}{4} = 5/4 = m$

$y(4) = 4 + \sqrt{4} = 6$

So $m = 5/4$ $(4, 6)$

$y - 6 = \frac{5}{4}(x - 4)$

$y = \frac{5}{4}x + 1$

[32pt] 4. Compute the derivative of the following functions:

(a) $f(x) = 2x^2 - \cancel{\frac{5}{x}} + \cancel{\sqrt{x}}$ $\quad 5x^{-1} + x^{1/2}$

Answer: $f' = 4x - 5/x^2 + 1/2\sqrt{x}$

$f' = 4x - 5x^{-2} + \frac{1}{2}x^{-1/2}$

$= 4x - \frac{5}{x^2} + \frac{1}{2\sqrt{x}}$

(b) $f(x) = \sqrt{3x^3 - x}$ $= (3x^3 - x)^{1/2}$

Answer: $f' = \frac{9x^2 - 1}{2\sqrt{3x^3 - x}}$

$f' = \frac{1}{2}(3x^3 - x)^{-1/2}(9x^2 - 1)$

(c) Use the product rule to compute : $f(x) = (x^3 + 3\sqrt{x})(x^2 - 5)$
(You do not need to simplify.)

Answer: $(x^3 + 3\sqrt{x})(2x)$ [redacted] $+ (3x^2 + \frac{3}{2}x^{-1/2})(x^2 - 5)$

$f' = (x^3 + 3\sqrt{x})(2x)$
$+ (3x^2 + \frac{3}{2}x^{-1/2})(x^2 - 5)$

(d) $f(x) = \dfrac{x^2}{4x^3 - 8}$ (Simplify your answer)

$$f' = \frac{(4x^3-8)(2x) - (12x^2)(x^2)}{(4x^3-8)^2}$$

$$= \frac{8x^4 - 16x - 12x^4}{(4x^3-8)^2}$$

Answer: $\dfrac{-4x^4 - 16x}{(4x^3-8)^2}$

5. An object moves along a straight line in such a way that at time t its position is given by the formula

$$s(t) = 2t^3 - 21t^2 + 60t - 25; \; 1 \le t \le 6$$

[8pt] (a) Find the velocity and the acceleration of the object at time t.

$$\boxed{\begin{aligned} v(t) &= s'(t) = 6t^2 - 42t + 60 \\ a(t) &= v'(t) = 12t - 42 \end{aligned}}$$

Answer:

[10pt] (b) Determine when the object is stationary, when it is moving forward, when it is moving backward on the given interval.

Answer:

$v(t) = 0$?

$6t^2 - 42t + 60 = 0$

$t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$

Stationary at $t = 5$ & $t = 2$

$v(1) = 6 - 42 + 60 = +$

$v(3) = 6(9) - 42(3) + 60 = -$

$v(6) = 6(36) - 42(6) + 60 = +$

$\boxed{\begin{array}{ll} \text{forward} & (1,2) \text{ \& } (5,6) \\ \text{Backward} & (2,5) \end{array}}$

[8pt] 6. Let $f(x) = x^2 - \sqrt{x}$. Find $f''(x)$.

$f' = 2x - \frac{1}{2} x^{-1/2}$

$f'' = 2 + \frac{1}{4} x^{-3/2}$

Answer: $f'' = 2 + \dfrac{1}{4\sqrt{x^3}}$