Key

Math 108 Exam 3 Spring 2001 Dr. Shapiro

Work carefully and neatly and remember that I cannot grade what I cannot read. You must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Notes and books are NOT ALLOWED. You may leave final answers using e and $\ln a$

[10] 1. Find the absolute maximum and minimum value of the function $f(x) = -3x^4 + 8x^3 - 10$ on the interval $-1 \leq x \leq 1$.

$f'(x) = -12x^3 + 24x^2 = 0$

$-12x^2(x-2) = 0$

$x = 0 \quad x = 2 \leftarrow$ not on interval

$f(-1) = -3 - 8 - 10 = -21$

$f(0) = -10$

$f(1) = -3 + 8 - 10 = -5$

max value $= -5$

min value $= -21$

[12] 2. Sketch the graph of the function $f(x) = \dfrac{x}{x^2 - 4}$. Be sure to include all asymptotes.

V.A. $x^2 - 4 = 0$

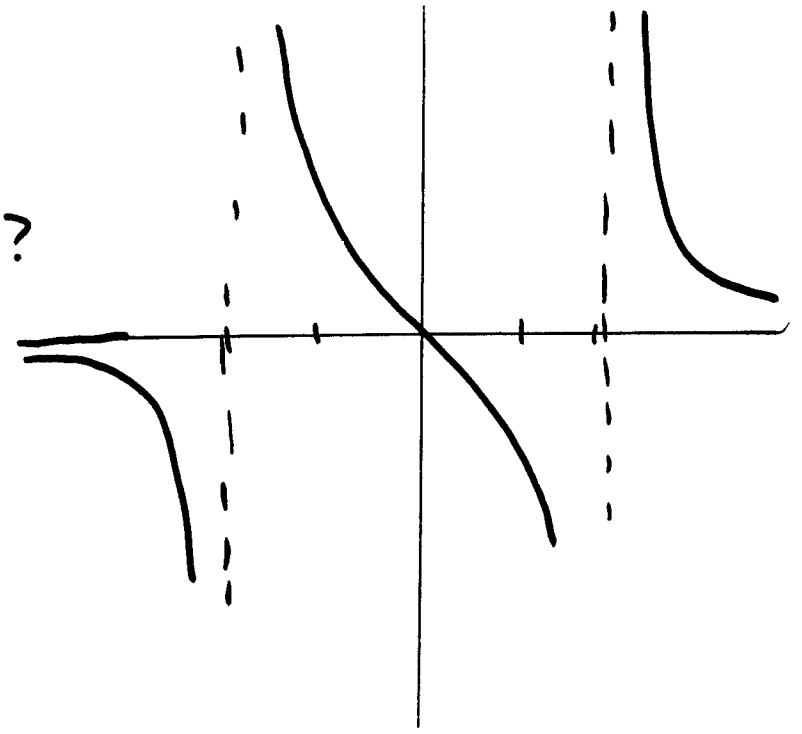
$x = \pm 2$

H.A. $y = 0$

$f'(x) = \dfrac{-x^2 - 4}{(x^2-4)^2} = 0$?

never

x	y
0	0
1	−
−1	+
3	+
−3	−



[8] 3. A bank which gives continuous compounding, promises to double your money in 7 years. What interest rate is the bank giving?

$7 = \dfrac{\ln 2}{r} = \dfrac{70}{r}$

$r = \dfrac{70}{7} = 10\%$

[16] 4. In the following solve for x:

(a) $e^{4x-2} = 7$.

$\ln \quad 4x - 2 = \ln 7$

$4x = \ln 7 + 2$

$x = \frac{1}{4}(\ln 7 + 2)$

(b) $5 = 3\ln x - \frac{1}{2}\ln x$

$= \frac{5}{2}\ln x$ | $e^2 = x$

$5 = \frac{5}{2}\ln x$

$2 = \ln x$

[8] 5. If $\ln a = 3.5$ and $\ln b = 4.6$, find $\ln \frac{\sqrt{b}}{a^3}$.

$\ln \frac{\sqrt{b}}{a^3} = \ln b^{1/2} - \ln a^3$

$= \frac{1}{2}\ln b - 3\ln a = \frac{1}{2}(4.6) - 3(3.5)$

$= -8.2$

[10pts] 6. In 1968 the value of a certain Van Gogh painting was \$2,000,000. In 1978 the value of the painting is \$3,000,000. Assuming the value grows exponentially, what will the value of the painting be in 1988?

$Q = 2e^{kt}$

$t=2 \quad 3 = 2e^{10k}$

$\frac{3}{2} = e^{10k}$

$Q(20) = 2e^{20k} = 2(e^{10k})^2$

$= 2\left(\frac{3}{2}\right)^2 =$

$2 \cdot \frac{9}{4} = \frac{9}{2}$ million

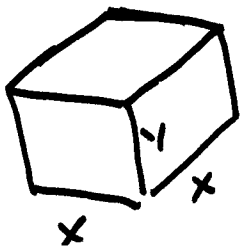
$= 4,500,000$

[6pts] 7. If you invest \$5,000 at 8% compounded quarterly, how much money will you have in 5 years?

$5,000\left(1 + \frac{.08}{4}\right)^{20}$

$= 5000(1.02)^{20}$

[10] 8. An box with an open top and square base is to be constructed so that its volume is 27 in^3. The cost of making the sides is 2 cents per square inch, while the cost of making the bottom is 4 cents per square inch. What dimensions of this box will minimize the cost?



$V = 27 = x^2 y$

$C = 2(4xy) + 4x^2$

$= 8xy + 4x^2$

$y = \frac{27}{x^2}$ $= 8x\left(\frac{27}{x^2}\right) + 4x^2$

$= \frac{8(27)}{x} + 4x^2 = C$

$C' = 8(27)(-x^{-2}) + 8x$

$= -\frac{8(27)}{x^2} + 8x = 0$

$8x = \frac{8 \cdot 27}{x^2}$

$x^3 = 27$

$x = 3, y = 3$

[10] 9. An antique store can obtain copies of a certain candle holder at cost of \$5 apiece. The store has been selling the holders at \$10 apiece and, at this price has been selling 15 holders per month. The store is planning to lower the price to stimulate sales and estimates that for each \$1 reduction in price, 5 more holders will be sold each month. At what price should the candle holders be sold in order to maximize monthly profit?

$P = (\#\text{ of item sold}) \times (\text{profit per item})$

$= (15 + 5x)(10 - x - 5)$

$= (15 + 5x)(5 - x)$

$= -5x^2 + 10x - 75$

$P' = -10x + 10$

$x = 1$

$x = \#$ of \$1 reduction

price $= 10 - 1$

$= 9$

[10] 10. A radioactive substance decays exponentially at an annual decay rate of $k = .002$. Currently there are 60 grams of the substance. In how many years will there be 20 grams of the substance. (You may leave an answer involving a logarithmic expression.)

$Q = 60e^{-.002t}$

$20 = 60e^{-.002t}$

$\frac{1}{3} = e^{-.002t}$

$\ln \frac{1}{3} = -.002t$

$t = \frac{-\ln \frac{1}{3}}{.002}$