Math 108 **Exam 2** **Spring 2001** **Dr. J. Shapiro**

Work carefully and neatly and remember that I cannot grade what I cannot read. You must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Graphing calculators are NOT allowed

[36pts] 1. Compute the derivative of the following functions:

(a) $f(x) = x^3 - x^{1/3} + \frac{1}{x^2}$ x^{-2}

$f' = 3x^2 - \frac{1}{3}x^{-2/3} - 2x^{-3}$

Answer: $f' = 3x^2 - \frac{1}{3}x^{-2/3} - 2x^{-3}$

(b) $f(x) = \sqrt{3x^2+1} = (3x^2+1)^{1/2}$

$f' = \frac{1}{2}(3x^2+1)^{-1/2}(6x)$

Answer: $f' = 3x(3x^2+1)^{-1/2}$

(c) $f(x) = 2x^2(x^2-2)^7$

$f' = 2x^2[7(x^2-2)^6(2x)]$
$+ 4x(x^2-7)^2$
$= 28x^3(x^2-2)^6 + 4x(x^2-7)^2$

Answer: $= 28x^3(x-2)^6 + 4x(x^2-7)^2$

(d) $f(x) = \dfrac{2x}{x^3-2}$

$f' = \dfrac{(x^3-2)(2) - 2x(3x^2)}{(x^3-2)^2}$

$= \underline{2x^3 - 4 - 6x^3}$

Answer: $= \dfrac{-4x^3-4}{(x^3-2)^2}$

[24pts] 2. The gross annual earnings of a certain company were $A(t) = .1t^2 + 10t + 20$ thousand dollars t years after its formation in 1991.

(a) At what rate were the gross annual earnings of the company changing with respect to time in 1995?

$A'(t) = .2t + 10$

$A'(4) = .2(4) + 10$

Answer: \$ 10.8 thousand

(b) At what percentage rate where the gross annual earnings growing with respect to time in 1995?

$$\frac{100\,A'(t_0)}{A(t_0)}$$

$A(4) = .1(16) + 10(4) + 20 = 61.6$

$$\frac{100\,[10.8]}{61.6}$$

Answer: $\frac{100\,[10.8]}{61.6} = 17.5\%$

(c) In what year will the rate of change of the population be 20 thousand people per year?

Set $A'(t) = 20$

$.2t + 10 = 20$

$.2t = 10$

$t = \frac{10}{.2} = 50$

Answer: year 2041

[12pts] 3. Draw the graph of $f(x) = x^4 + 4x^3$. Be sure to include all critical values and relative extrema.

$f'(x) = 4x^3 + 12x^2$

$= 4x^2(x+3)$

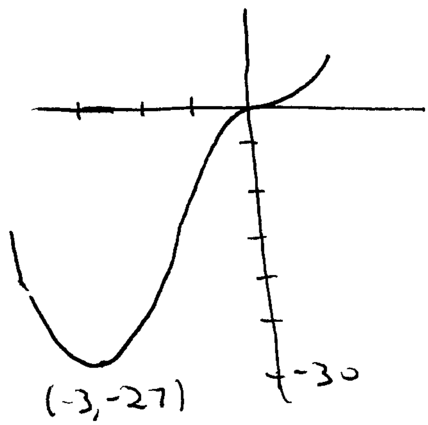
C.V. $x = 0$, $x = -3$

$f'(-4) = (+)(-) = -$

$f'(-1) = +(+) = +$

$f'(1) = +(+) = +$

x	y
-3	-27
0	0



(-3, -27)

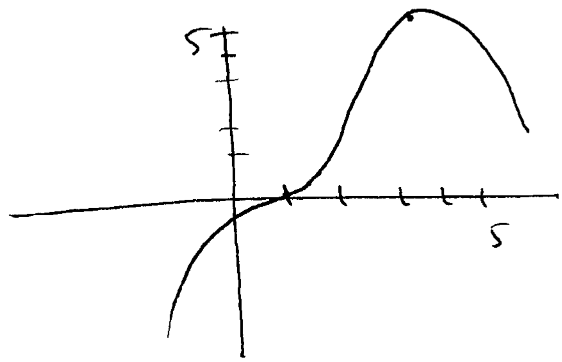
rel. min

[10pts] 4. Draw the graph of a function $f(x)$ with the following properties:

(a) $f(1) = 0$, $f(4) = 5$;

(b) $f'(1) = f'(4) = 0$;

(c) $f'(x) > 0$ when $x < 4$, and $f'(x) < 0$ when $x > 4$.



[8pts] 5. Find the point(s) of inflection of the function $f(x) = (x+1)^{2/3}$, then determine where it is concave up and where it is concave down.

Answer:
f concave down on real line

$f' = 2/3\,(x+1)^{-1/3}$

$f'' = -\frac{2}{9}(x+1)^{-4/3} = \frac{-2}{9(x+1)^{4/3}}$

note: $(x+1)^{4/3}$ is always positive

Hence $f'' < 0$ for all x

So f is concave down

[10pts] 6. A manufacturer's total monthly revenue is $R(q) = 240q + 0.05q^2$ dollars when q units are produced during the month. Currently, the manufacturer is producing 80 units a month and is planning to decrease the monthly output by .5 units. Estimate how the total monthly revenue will change as a result.

Answer:
-124 dollars

$\Delta R(q) \simeq R'(q_0)\,\Delta q$

$q_0 = 80$

$\Delta q = -.5$

$R'(q) = 240 + .1q$

$R'(80) = 240 + .1(80) = 248$

$\Delta R(q) \approx (248)(-.5)$

$= -124$