HW9

Problem 1. A differentiable function $f: R^n \to R$ is called strongly convex if for all $x, y \in R^n$ we have

$$f(y) \geq f(x) + \langle \nabla f(x), y - x \rangle + \frac{m}{2} \|y - x\|^2$$

for some $m > 0$. For a strongly convex differentiable function $f: R^n \to R$ with the strong convexity constant m (definition is above) prove the following inequality:

$$f(x) \geq f(x^*) + \frac{m}{2} \|x - x^*\|^2$$

for all $x \in R^n$, where x^* is a minimizer of $f(x)$.

Problem 2. For a strongly convex differentiable function $f: R^n \to R$ with the strong convexity constant m (definition is above) prove the following inequality:

$$\langle \nabla f(x) - \nabla f(y), x - y \rangle \geq m \|x - y\|^2$$

for all $x, y \in R^n$.

Problem 3. For a strongly convex differentiable function $f: R^n \to R$ with the strong convexity constant m (definition is above) prove the following inequality:

$$\|\nabla f(x) - \nabla f(y)\| \geq m \|x - y\|$$

for all $x, y \in R^n$ (Hint use the result of Problem 2).

Problem 4. Using paper and pencil:

Determine the minimizers/maximizers of the following functions:

(i) $f(x_1, x_2) = x_1^4 + x_2^4 - 4x_1x_2$.

(ii) $f(x_1, x_2) = x_1^2 - 2x_1x_2^2 + x_2^4 - x_2^5$.

(iii) $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 5x_3^2 - 2x_1x_2 - 4x_2x_3 - 2x_3$.