HW9

1. Implement nonlinear rescaling-augmented Lagrangian (NRAL) method and solve the following problems:

$$\min 3x_1 + 2x_2 + x_3 + x_4,$$
$$\text{s.t. } x_1^2 + x_2^2 + x_3^2 + x_4^2 \le 10,\ x_1 + x_2 + 2x_3 + 3x_4 = 1,\ x_1 \ge 1.$$

$$\min x_1 + x_2^2 + x_3 + x_4 + x_5^4,$$
$$\text{s.t. } x_1^2 + x_2^2 + x_3^2 + x_4^2 \le 9,\ x_1 + 2x_2 + 2x_3 + 4x_4 = 1,\ 2x_3 - 4x_4 = 1,\ x_1^2 - x_5 \le 2,$$
$$x_4 \ge 0.5.$$

For the stopping rule of each unconstrained minimization use the criteria:
$\left\|\nabla_x \Phi(x^{s+1}, y^s, z^s)\right\| \le \max\left\{\frac{1}{k}\left\|y^s - \Psi'\left(kC(x^{s+1})\right)y^s\right\|, \left\|g(x^{s+1}), \right\|10^{-6}\right\}$.

For the stopping rule of the NRAL method use the criteria:
$\max\left\{\left\|\nabla_x L(x^s, y^s, z^s)\right\|, \left\|Y^s c(x^s)\right\|, \max_{1\le i\le m}\{-c_i(x^s)\}, \left\|g(x^s)\right\|\right\} \le 10^{-6}$. Perform minimization using Newton's method. Record the number of Newton iterations for each unconstrained minimization between updates and $\max\left\{\left\|\nabla_x L(x^s, y^s, z^s)\right\|, \left\|Y^s c(x^s)\right\|, \max_{1\le i\le m}\{-c_i(x^s)\}, \left\|g(x^s)\right\|\right\}$ after each Lagrange multipliers update. Use $k = 100$ and $\eta = 0.1$ (parameter for Armijo rule).