1. Using paper and pencil:

Determine the minimizers/maximizers of the following functions:

(i) \( f(x_1, x_2) = x_1^4 + x_2^4 - 4x_1x_2. \)
(ii) \( f(x_1, x_2) = x_1^2 - 2x_1x_2^2 + x_2^4 - x_2^2. \)
(iii) \( f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 5x_3^2 - 2x_1x_2 - 4x_2x_3 - 2x_3. \)

2. For a strongly convex differentiable function \( f: \mathbb{R}^n \to \mathbb{R} \) with the strong convexity constant \( m \) (definition in slide 14, inequality (33)) prove the following inequality:

\[
\langle \nabla f(x) - \nabla f(y), x - y \rangle \geq m\|x - y\|^2.
\]

3. Consider the gradient descent method for finding a minimum of a differential convex function \( f(x) \) defined on \( \mathbb{R}^n, \ n \geq 2 \). Assume that the minimum exist. Suppose that at each step (iteration), the method selects the step length (step size) using one-dimensional minimization of \( f(x) \) along its direction of descent. Prove that, in case of more than one step (iteration), two consecutive descent directions are orthogonal to each other.

4. (Extra credit) Consider a strongly convex function differentiable function \( f(x) \) defined on \( \mathbb{R}^n \) with the strong convexity constant \( l \). Prove that for all \( x \in \mathbb{R}^n, \ y \in \mathbb{R}^n \) the following inequality holds:

\[
f(y) \leq f(x) + \langle \nabla f(x), y - x \rangle + \frac{1}{2l}\|\nabla f(x) - \nabla f(y)\|^2.
\]