Homework 1 (due February 3)

Formulate problems 1-11 in ampl and solve numerically. Show the ampl models and the optimal solution.

1. $\min 100(x_2 - x_1^2)^2 + (1 - x_1)^2,\ \text{s.t.}\ x_2 \geq -1.5.$

2. $\min 100(x_2 - x_1^2)^2 + (1 - x_1)^2,\ \text{s.t.}\ x_2 \geq 1.5.$

3. $\min x_2 + 0.00001(x_2 - x_1)^2,\ \text{s.t.}\ x_2 \geq 0.$

4. $\min (x_1 + 1)^3 / 3 + x_2,\ \text{s.t.}\ x_1 \geq 1, x_2 \geq 0.$

5. $\min \sin(x_1 + x_2) + (x_1 - x_2)^2 - 1.5x_1 + 2.5x_2 + 1,\ \text{s.t.} -1.5 \leq x_1 \leq 4,\ -3 \leq x_2 \leq 3.$

6. $\min (1 - x_1)^2,\ \text{s.t.} 10(x_2 - x_1^2) = 0.$

7. $\max x_2 - \log(x_1^2 + 1),\ \text{s.t.}\ (x_1^2 + 1)^2 + x_2^2 = 4.$

8. Find any (x_1, x_2) such that $x_1^{\ 2} + x_2^2 = 25, x_1 x_2 = 9.$

9. $\min \sin(\pi x_1 / 12)\cos(\pi x_2 / 16),\ \text{s.t.}\ 4x_1 - 3x_2 = 0.$

10. $\max x_2 - x_1,\ \text{s.t.} -3x_1^2 + 2x_1 x_2 - x_2^2 \geq -1.$

11. Find position of $n = 2$ and $n = 20$ electrons on a sphere of radius $R = 1$ by solving the following problem:

$$\min \sum_{i=1}^{n} \sum_{j>i} \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}},$$
$$\text{s.t.}\ \ x_i^2 + y_i^2 + z_i^2 \leq R^2,\ i = 1,\ldots,n.$$

Make sure to provide a good initial guess for this problem: no 2 electron can be at the same initial location, otherwise denominators may become zero. Use ampl "let" command to provide the initial guess.