Homework 1

Solve numerically the following problems. Show the models and optimal solutions.

1. \( \min 100(x_2 - x_1^2)^2 + (1 - x_1)^2, \) s.t. \( x_2 \geq -1.5. \)

2. \( \min x_2 + 0.00001(x_2 - x_1)^2, \) s.t. \( x_2 \geq 0. \)

3. \( \min (x_1 + 1)^3 / 3 + x_2, \) s.t. \( x_1 \geq 1, x_2 \geq 0. \)

4. \( \min \sin(x_1 + x_2) + (x_1 - x_2)^2 - 1.5x_1 + 2.5x_2 + 1, \) s.t. \( -1.5 \leq x_1 \leq 4, -3 \leq x_2 \leq 3. \)

5. \( \min (1 - x_1)^2, \) s.t. \( 10(x_2 - x_1^2) = 0. \)

6. \( \max x_2 - \log(x_1^2 + 1), \) s.t. \( (x_1^2 + 1)^2 + x_2^2 = 4. \)

7. \( \min \sin(\pi x_1 / 12) \cos(\pi x_2 / 16), \) s.t. \( 4x_1 - 3x_2 = 0. \)

8. \( \max x_2 - x_1, \) s.t. \( -3x_1^2 + 2x_1x_2 - x_2^2 \geq -1. \)

9. Find position of \( n = 2 \) and \( n = 20 \) electrons on a sphere of radius \( R = 1 \) by solving the following problem:

\[
\min \frac{1}{\sum_{i=1}^{n} \sum_{j>i} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}},
\]

s.t. \( x_i^2 + y_i^2 + z_i^2 \leq R^2, \) \( i = 1, \ldots, n. \)

Make sure to provide a good initial guess for this problem: no 2 electron can be at the same initial location, otherwise some denominators may become zero. Use ampl “let” command to provide the initial guess.