Computer assignment 3, MATH 114
Due November 8, 2012.

1. Consider the initial value problem

$$y' = y + t^2, \quad y(0) = 1.$$

Consider Euler's method for solving this problem for $t \in [0, 2]$.

a) Solve the problem with $\Delta t = 0.1$. Plot the numerical solution together with the exact solution $y = 3e^t - t^2 - 2t - 2$.

b) What should be a choice of the step Δt that guarantees finding the solution with an accuracy 10^{-2}? Find the Lipschitz constant L. Use $M = 3e^2 - 2$ such that $|y''(t)| \leq M$ for all $t \in [0, 2]$.

c) Implement Euler's method and solve the problem using Δt from (b). Compare the accuracy of the obtained solution with the exact solution $y = 3e^t - t^2 - 2t - 2$. Plot $|y(t) - w(t)|$, where $w(t)$ is a numerical solution. What is $\max\limits_{0 \leq t \leq 2} |w(t) - y(t)|$?