Subcritical case.

Trajectories that start on the upper boundary of the rectangle in the right half of the rectangle:

$$\rho^0(y) = \frac{3c}{8(a-2\kappa)} \cdot \frac{e^{-\frac{y^2}{2s^2\tau}}}{s\sqrt{2\pi\tau}}. \tag{1}$$

Trajectories that start in the right half of the rectangle, but not on the upper boundary, and do not hit the right boundary of the rectangle:

$$\rho^1(y) = \int_y^{c/2} \rho_1(y,m) \left[\left(\frac{c}{2} - m\right) \frac{a-c-2\kappa}{c(a-2\kappa)} + \left(\frac{c^2}{4} - m^2\right) \frac{1}{2c(a-2\kappa)} \right] dm, \qquad 0 < y < c/2,$$

$$\rho^1(y) = \int_0^{c/2} \rho_1(y,m) \left[\left(\frac{c}{2} - m\right) \frac{a-c-2\kappa}{c(a-2\kappa)} + \left(\frac{c^2}{4} - m^2\right) \frac{1}{2c(a-2\kappa)} \right] dm, \qquad y < 0.$$

Note that two different integrals describe the density for positive and negative y. Here

$$\rho_1(y,m) = \frac{2(2m-y)}{s^3\tau\sqrt{2\pi\tau}} e^{-\frac{(2m-y)^2}{2s^2\tau}}, \quad m \geq 0, m \geq y.$$

Trajectories that reach the upper boundary of the rectangle: I think, we established that these trajectories are unlikely for the subcritical case.

Trajectories (starting from the right half of the rectangle) that hit the right boundary of the state rectangle, but do not hit the upper boundary:

$$\rho^4(y) = \rho_* \frac{A}{2} \int_{\max\{0,2(y-c/2)/(a-c)\}}^2 (2-p)\, dp \int_{\max\{0,y-p(a-c)/2\}}^{c/2} \rho_1(y-\kappa p, q+Ap/2)\, dq$$
$$+\frac{A}{2} \int_{\max\{0,2(y-c/2)/(a-c)\}}^2 dp \int_{\max\{0,y-p(a-c)/2\}}^{c/2} \frac{q}{c(a-2\kappa)} \rho_1(y-\kappa p, q+Ap/2)\, dq \tag{2}$$

Here

$$A = a - c - 2\kappa,$$
$$\rho_* = \frac{a-c-2\kappa}{2c(a-2\kappa)}.$$

In the *subcritical case* we need to sum (1) and all the following integrals. The sum $\rho(y)$ then needs to be symmetrized (to take care of the trajectories starting in the left part of the rectangle) so that

$$\rho(y) + \rho(-y)$$

is the analytic approximation to the *density* function of the daily increments of the log-price.