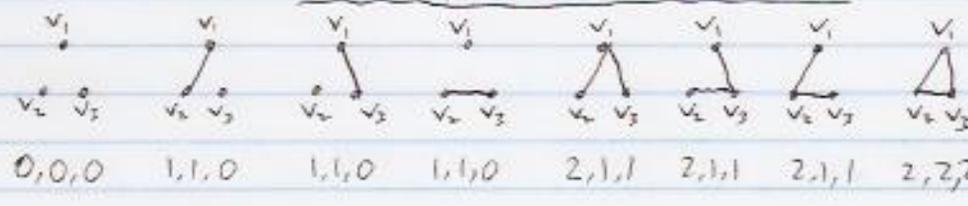


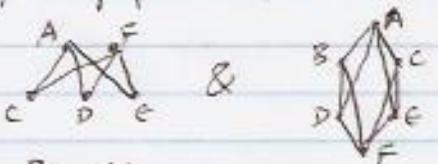
## §9.2 Homework Solutions

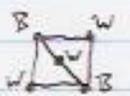
- 2)  Not sensible for pseudographs, which may have arbitrarily many edges.

- 3) [BB]      6) a)  $n-1$     b)  $\binom{n}{2} = \frac{n(n-1)}{2}$     c) Yes  $K_n$

- 14) i), iv), v) are all bipartite and are subgraphs. ii) and iii) are not bipartite and so cannot be subgraphs.

- 15) a) [BB]      b) No, left graph has 2 vertices of degree  $\geq 3$ , right graph has only 1.

- c) Yes, for example  d) No, all triangles on RHS have a common vertex.

- 21) a) c) [BB]    b) Yes  d) No, there exists an odd cycle (length 5)

- e) Yes      f) No, there exists a 3-cycle.

- 23) a) Yes    b) Yes it would change

- 25) They are  $K_{0,n}, K_{1,n-1}, K_{2,n-2}, \dots, K_{\frac{n}{2}, \frac{n}{2}}$  if  $n$  even, so answer is  $\frac{n}{2} + 1$  if  $n$  even  
 $K_{0,n}, K_{1,n-1}, \dots, K_{\frac{n-1}{2}, \frac{n-1}{2}}$  if odd, so answer is  $\frac{n+1}{2}$  if  $n$  odd

- 26) a)  $2^{\binom{n}{2}}$     b)  $2^{\binom{n}{2}-3}$     c)  $\binom{n}{3} 2^{\binom{n}{2}-3}$     d)  $\frac{\binom{n}{3} 2^{\binom{n}{2}-3}}{2^{\binom{n}{2}}} = \frac{\binom{n}{3}}{8}$

- 28) Let  $n = \#$  vertices. Then sum of degrees  $\geq 3n$ . But ~~sum~~ sum of degrees = 70  
 So  $70 \geq 3n \Rightarrow n \leq 23$ .

- 35) The 10 vertices we know about have degree sum  $3+28=31$  so the remaining 3 vertices with unknown degree must also sum to 31 (since total degree sum =  $2 \times 31 = 62$ ).

The highest possible degree of each of these <sup>three</sup> is  $12 \times 3 = 36$  but each of the degree 1 vertices can only be connected to at most one of them. So highest possible total of unknown vertices is  $36 - 3 \times 2 = 30$ .

So it is not possible to find such a graph