§7.2 Homework Solutions

3) [BB]

7) a) $\binom{49}{6}$ b) $\binom{49}{6} = 13,983,816$ so you need to fill at least 14 cards

c) $\binom{43}{6}, \binom{6}{1}\binom{43}{5}, \binom{6}{2}\binom{43}{4}, \binom{6}{3}\binom{43}{3}, \binom{6}{4}\binom{43}{2}, \binom{6}{5}\binom{43}{1}, \underbrace{\binom{6}{6}\binom{43}{0}}_{=1}$ (in that order)

d) The answer to a) ie. $\binom{49}{6}$ since the events in c) are mutually exclusive but one of them must happen.

e) $\binom{49}{6} - \binom{43}{6}$ f) $\binom{49}{6} - \binom{43}{6} - \binom{6}{1}\binom{43}{5}$

11) [BB] Note that the balls are numbered and so distinguishable but the order is not important which is why it is a combination and not a permutation.
If the red balls were identical and the white balls were identical see §7.5.

14) a) $\binom{18}{4}$ (the strongest and weakest must be chosen and the remaining 4 come from 18)
b) $\binom{18}{5}$ (the strongest must be picked and then 5 more from a possible 18)
c) $\binom{18}{6}$

20) a) Any 3 vertices define a triangle, so ~~$\binom{20}{3}$~~ $\binom{12}{3}$

b) There are 12×8 triangles that have exactly 1 edge in common with the polygon (8 for each edge of the polygon).
There are also 12 triangles that have 2 edges in common
So Answer is $\binom{12}{3} - 12 \times 8 - 12$

25) a) RHS $= \frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-k-1)!} = \frac{k(n-1)! + (n-k)(n-1)!}{k!(n-k)!} = \frac{n(n-1)!}{k!(n-k)!} =$ ~~$\frac{n!}{k!(n-k)!}$~~

$= \frac{n!}{k!(n-k)!} = \binom{n}{k} =$ LHS ✓

b) Imagine a set with n objects and we need to choose k.
This can be done in $\binom{n}{k}$ ways

Now select a special element at random from the set.
$\binom{n-1}{k}$ subsets do not contain this set and $\binom{n-1}{k-1}$ do (see Question 14)
So $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ by the addition rule.