§5.1 Homework Solutions

1) abc [BB] d) $9+27+81=117$ e) $4+1+2+7+16+29=59$ f) $1-1+1-1+\dots+(-1)^n$
$= \begin{cases} 0 & n \text{ even} \\ 1 & n \text{ odd} \end{cases}$

4) a) [BB] d) Step 2: Check $n=1$: $8^1-3^1=5$ which is a multiple of 5 ✓

Step 1. Assume P_k is true, i.e. $8^k-3^k \in 5\mathbb{Z}$ for arbitrary $k \geq 1$

Consider $8^{k+1}-3^{k+1} = 8\times 8^k - 3\times 3^k = (3+5)8^k - 3\times 3^k = 3(8^k-3^k)+5\times 8^k$

By the inductive hypothesis this is the sum of two multiples of 5 So P_{k+1} is true □

f) [BB]

7) a) [BB], b) $\sum_{i=1}^{n}(-1)^{i+1}i^2$ c) $\sum_{i=1}^{n}(2i-1)^2$ d) $\sum_{i=1}^{n} i(i+1)(i+2)$ e) $\sum_{i=1}^{n}\frac{1}{i(i+1)}$

9) e) Choose any $x>-1$ and fix it.
Step 2: Check $n=1$. $(1+x)^1 = 1+1\times x$ ✓
Step 1: Assume the result holds for k i.e. $(1+x)^k \geq 1+kx$ (the inductive hypothesis)
Then $(1+x)^{k+1} = (1+x)^k(1+x) \geq (1+kx)(1+x)$ (by the inductive hypothesis)
$= 1+kx+x+kx^2 \geq 1+(k+1)x$

So the result holds for $k+1$. □

h) Step 2. The result holds for $k=2$ since $1+\frac{1}{\sqrt{2}} \geq \sqrt{2}$ ✓

Step 1. Assume the result holds for some arbitrary $k \geq 2$
$\Rightarrow 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots+\frac{1}{\sqrt{k}} \geq \sqrt{k}$
$\Rightarrow \left(1+\frac{1}{\sqrt{2}}+\dots+\frac{1}{\sqrt{k}}\right)+\frac{1}{\sqrt{k+1}} \geq \sqrt{k}+\frac{1}{\sqrt{k+1}}$ (by the inductive hypothesis)
$= \frac{\sqrt{k}\sqrt{k+1}+1}{\sqrt{k+1}} \geq \frac{k+1}{\sqrt{k+1}} = \sqrt{k+1}$

So the result holds for $k+1$ □

11) [BB] 12) [BB] 15) [BB].