§3.3 Homework Solutions

2) The following one-to-one correspondence $\{1, 2, 3, 4, \ldots\}$ $\updownarrow\updownarrow\updownarrow\updownarrow$ $\{1, 4, 9, 16, \ldots\}$ shows the two sets have the same cardinality.

6) This is only true if B is finite, but may be false if B is infinite

9) Let $a \in A$ and $b \in B$. Then $f : (a, b) \mapsto (b, a)$ is a bijection between $A \times B$ and $B \times A$. So $|A \times B| = |B \times A|$.

11) a) Let $\sim$ mean "has the same cardinality".
Since Identity function is a bijection from $A \to A$, we have $A \sim A$
The inverse of any bijection exists and is also a bijection, so $A \sim B \Leftrightarrow B \sim A$.
The composition of two bijections is also a bijection. Thus $A \sim B$ & $B \sim C \to A \sim C$.
Thus Cardinality is an equivalence relation.

b) [BB] c) [BB] d) $f : x \mapsto x + b - a$ gives a 1-1 correspondence.

12) a) [BB] b) $f(x) = 2x + 4$ defines a one-to-one correspondence.

c) $f(x) = \left(\frac{d-c}{b-a}\right)x + \underbrace{c - \frac{ad-ac}{b-a}}$ (simplify this if you like)

stretches (a, b) to be the same length as (c, d)

& shifts interval so that it matches (c, d).

19) a) $\{2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096 \ldots\}$ b) $\{1, 2, \frac{1}{2}, 4, \frac{1}{4}, 8, \frac{1}{8}, 16, \frac{1}{16}, 32, \frac{1}{32}, 64, \frac{1}{64} \ldots\}$

c) $\{1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34 \ldots\}$ d) $\{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), \ldots\}$

e) [BB] f) Use Figure 3.8 with an extra row for 0
$\{(1,0), (2,0), (1,1), (1,-1), (1,2), (1,-2), (2,1), (2,-1), (3,0), (4,0), (3,1), (3,-1), \ldots\}$

g) Use Figure 3.8 with an extra row and column.
$\{(0,0), (1,0), (-1,0), (0,1), (0,-1), (0,2), (0,-2), (1,1), (1,-1), (-1,1), (-1,-1), (2,0), (-2,0), \ldots\}$

21) a) finite b) Countably infinite $\{0, 3, \frac{1}{3}, 9, \frac{1}{9}, \ldots\}$

c) Infinite (Since you can use colons etc to construct arbitrarily long sentences!)