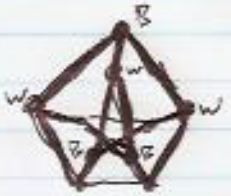
§13.1 Homework Solutions

1) [BB]

4) The one on the left is planar. For example it can be redrawn 

The Petersen graph is not planar because it contains a subgraph homeomorphic to $K_{3,3}$.



6) Since G is planar $V-E+R=2$

Define $N = \sum_{\text{regions}} \#\text{edges} \geq 5R$ since each region has ≥ 5 edges

But each edge included at ~~least~~ most twice in N, so $2E \geq N$

So $2E \geq 5R$ and $5V-5E+5R=10 \Rightarrow 5V-5E+2E \geq 10$

$\Rightarrow \underline{5V-10 \geq 3E}$

10) a) For $n=1,2,3,4$. b) $K_{1,n}$ and $K_{2,n}$ are always planar for any n since they do not contain $K_{3,3}$ or K_5.
Any other $K_{m,n}$ ~~where~~ $m \& n \geq 3$ so that $K_{3,3}$ is a subgraph. Therefore $K_{1,n}$ and $K_{2,n}$ are the only ones

21) a) [BB]

b) Remove the 7 degree 1 vertices and those 7 edges. Call ~~this~~ this graph G'.

G' is still planar and connected, since we only removed leaves.
$E' \leq 3V'-6 \Rightarrow E' \leq 3 \times 13 - 6 = 33$

But $E = E' + 7$ so $\underline{E \leq 40}$