§10.2 Homework Solutions

1) [BB]

2 a)  No, the 4 edges highlighted must be in any Hamiltonian cycle but already form a cycle.

b) [BB]

c)  No. D.H.o

d) [BB]

e) No, going around the outside forms a non-Hamiltonian cycle

f) ~~No~~ Yes, going around the hexagon is a Hamiltonian cycle.

5) a) [BB] b) Yes ABDCE (any Hamilton Cycle with any edge removed is a Hamiltonian Path!)

c) ~~Yes~~ No d) Yes, going between A and E since these are the only odd vertices.

9) a) [BB] b) Yes because the dual graph (see answer to a)) is Eulerian

15) a) [BB] b) Yes (easy to find one) c) Yes 

d) Both are false and  is a counterexample.

e)

23) K_{11} has $\binom{11}{2} = 55$ edges. So our graph is K_{11} with 2 edges missing.

Case i) both missing edges are adjacent to some vertex v.
Then every vertex has degree $\geq 8 \geq 11/2$ So Hamiltonian by Dirac's Theorem

But there are 2 odd vertices (degree 9) so not Eulerian.

Case ii) the missing edges are not adjacent.
Then every vertex has degree ≥ 9 so Hamiltonian by Dirac's Theorem

There are now 4 vertices of odd degree so not Eulerian

In either case, Hamiltonian but not Eulerian.