# §0.1 Homework Solutions

2) a) [BB]     b) False ∧ True  so FALSE     g) [BB]

j) A double implication $p \leftrightarrow q$ is true if $p$ and $q$ have the same truth value. $4 = 2+2$ and $7 < \sqrt{50}$ are both true so the statement is TRUE

k) TRUE since hypothesis is false     l) FALSE since $4 \neq 2+2$ is false but $7 < \sqrt{50}$ is true

m) FALSE since hypothesis true but conclusion false     n) [BB]

o) FALSE since hypothesis true but conclusion false     p) [BB]

q) The hypothesis is always false so the implication is TRUE

5) a) [BB]     b) $x$ is not a real number or $x^2 + 1 \neq 0$     c) [BB]

d) There exists an integer that is not divisible by a prime.     e) [BB]

f) For all $a, b, c$, $(ab)c = a(bc)$     g) [BB]

i) There exists an $x > 0$ such that $x^2 + y^2 \leq 0$ for some $y$.     k) [BB]     l) [BB].

6)     a) [BB]     b) Converse: $x = \pm 1 \rightarrow x^2 = 1$. Contrapositive: $x \neq \pm 1 \rightarrow x^2 \neq 1$.

e) [BB]     f) Converse: $a = 0$ or $b = 0 \rightarrow ab = 0$. Contrapositive: $a \neq 0$ and $b \neq 0 \rightarrow ab \neq 0$

g) Converse: A four-sided figure is a square. Contrapositive: A non-4 sided-figure is not a square
                                                                                h) [BB]

7) a) [BB]  b) $\forall$ real $x$, $2^x \geq 0$   c) [BB]  d) $\forall$ primes $p$, $\exists$ prime $q$ with $q > p$

e) [BB]  f) $\forall x > 0$, $\sqrt{x}$ is real

8)     No. Consider $p \rightarrow q$. If this is false then $p$ must be true and $q$ false. So the converse $q \rightarrow p$ must be (vacuously) true.