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A MESOSCOPIC STOCK MARKET MODEL WITH HYSTERETIC AGENTS

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Abstract. Following the approach of [22], we derive a system of Fokker-Planck equations to model a stock-market in which hysteretic agents can take long and short positions. We show numerically that the resulting mesoscopic model has rich behaviour, being hysteretic at the mesoscale and displaying bubbles and volatility clustering in particular.

1. Introduction.

In a series of publications [11, 9, 12, 20, 10, 19] we have developed a model of financial markets that, in common with a number of other models, reproduces the most important observed statistics of real-world financial systems. Such heterogeneous agent models are very convenient as they make explicit the assumptions on the psychology and motivations of traders (so that the plausibility of the assumptions can be compared across models) and they are also relatively easy to program. On the other hand, a full and rigorous analysis of such models is well-nigh impossible. Faced with this difficulty, in recent years much effort has been expended in deriving mesoscopic versions of heterogeneous agent models (see [7], for example). In brief, if the agents are structured by a variable s, mesoscopic models describe the evolution of $\rho(x, t)$, the density of agents having $s = x$ at time t. Many such models fit into Boltzmann-type kinetic theory and hence can use the mathematical tools developed for other applications of that theory [13]. In other words, these models assume that the values of the structuring variable s of a trader change as a result of a binary interaction with another agent. This framework is powerful and useful, for example, in discussions of wealth distribution evolution. However in a stock market context a different situation is more likely to occur,

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one in which all agents are exposed to an information stream and are also globally coupled via some measure of market *sentiment*.

A mesoscopic model along these lines, in which coupling between agents is "mimetic", i.e. due to herding, has been suggested by Omurtag and Sirovich [22], the final form of which is a single Fokker-Planck equation. In the present contribution, motivated by our earlier models we modify the approach of Omurtag and Sirovich in a number of respects, taking it further away from its origin in neuronal dynamics. To be more precise, we assume that our agents are *hysteretic* and can take long or short positions in the market. As in [22] we allow *global coupling via the public information stream.* We then proceed in two steps by first defining the individual dynamics of each trader. We then derive a system of two PDEs for the mesoscopic variables and simplify them to yield two coupled Fokker-Planck type equations that govern the evolution of the densities of long and short traders.

We dedicate this paper to the memory of Alexei Pokrovskii, who died unexpectedly on September 1, 2010. Apart from his seminal contributions to the mathematical analysis of systems with hysteresis, Alexei took a keen interest in applications to financial markets. The day before he died, he completed the first draft of [8], a combinatorial analysis that shows that arbitrage sequences in the foreign exchange (FX) market tend to be periodic in nature. The information streams that affect stock markets considered in the present paper also impact other markets such as FX [15]. Alexei would, no doubt, have pointed this out and made some lucid suggestions as to how the analysis in this paper could be extended to such market interactions. The high frequency data sets available for financial markets present rich opportunities for testing the implications of the hysteresis models pioneered by Alexei.

2. **Microstructure of stock markets.** The last two decades have witnessed a burgeoning literature on the microstructure of financial markets (see [4] for a survey). This interest has been spurred by a move away from traditional dealer markets towards electronic methods of executing transactions [16]. Much of the focus has been on how the adopted trading execution methods affect intra-day trading patterns, with empirical work dealing with high frequency, such as minute-by-minute, price and transactions data [17]. In relation to stock markets, a seminal study [1] found that the opening periods of trading on the New York Stock Exchange (NYSE) were characterised by a higher variance in stock returns than closing periods. This finding simulated interest in how market institutions and trading rules affect stock market outcomes.

Traditionally, stock markets were *dealer markets*. In such a system, coordination between buyers and sellers is provided by dealers who commit to buying and selling stocks at their quoted bid (buy) and ask (sell) prices. To provide this coordination function, dealers have to hold sufficient stock inventories to allow trades to be executed, the bid-ask price spread being seen as the return for providing this liquidity.

With the advent of electronic methods of executing transactions, *limit order markets* have come to play a dominant rôle in stock transactions. Here stock trades are coordinated by electronically matching orders, first by price, then by time of submission: for live order book data see `http://data.inetats.com/ds/tools/charts`. Buyers and sellers choose either limit orders, orders to buy or to sell when a given price is reached, or market orders to buy or sell at the current price in the limit order book. Some stock markets, such as the NYSE, are hybrid markets in that, although most of the trading is conducted via the electronic limit order book, each stock has a specialist dealer who quotes bid and ask prices for the specialist stock for trades up to a particular volume, the aim being to maintain a liquid market in the stock [21].

In the present paper it is assumed that traders face the same external information streams, none of this information being private. This assumption is more relevant to a limit order market in which the order flow information contained in the limit order book can be publicly observed. In a dealer market, where dealers receive private information on the order flows forthcoming at their quoted bid and ask prices, information sets will tend to be disjoint.

The end–users in a particular market will have access to private information streams arising from their own economic activities, as well as to the public information coming from, for example, the release of macroeconomic statistics by government or statistical agencies. In markets with fragmented methods for executing transactions, this end–user private information can be translated into private order flow information for trades. In the FX market, for example, 21.6% of the global turnover in 2010 was executed by customer–direct transactions [24, table E.24], giving the FX dealers involved private order flow information. Analysing the effects of the arrival of macroeconomic news at five–minute intervals during FX trading days, one estimate is that around two thirds of the price impact is transmitted by order flow information, the remaining one third being the direct impact of the news [14].

In the stock market the end–users are the private individuals who own stocks, often via pension-, life-, hedge-, and other funds, and the firms who issue stocks. All have potential access to private information streams. The question is then whether the traders operating on their behalf can exploit this private information. In limit order stock markets such as Hong Kong, Tokyo, Toronto, EURONEXT and INET, private information becomes public once the buy or sell orders are placed in the limit order book (see [3]), so providing an approximation to the assumption regarding information streams used in the present paper. In NYSE, London, Frankfurt, NASDAQ, MATIF, XETRA, and EUREX stock markets, where dealer market elements are still present, some of the information streams will remain private until the price impact can be observed.

3. **Individual dynamics.** We consider a market with a fixed number N of traders who can be either long or short on a stock. As in [22], we assume that the description of a trader involves a "propensity for action" variable s. For traders that are long, the variable s takes values in $(-1, \infty)$ and for ones that are short, it takes values in $(-\infty, 1)$. When s lies in the interval $(-1, 1)$ the trader can be either long or short, depending on whether this interval has been approached from above or from below. This "lazy relay" is illustrated in Figure 1.

One obvious reason for the distinct thresholds illustrated in Figure 1 is the existence of commission fees payable to brokers for executing trades. Depending on the institutional context, other components of the bid-ask price spread have been associated with inventory costs and asymmetric information amongst traders [4, 17]. Less obvious are the implicit costs associated with deliberations as to whether to be long or short. There is some evidence of thresholds in the neural processes linking perception to action [5], and the decision-making processes of investment funds will usually involve costs sunk in analysis and deliberation that cannot be

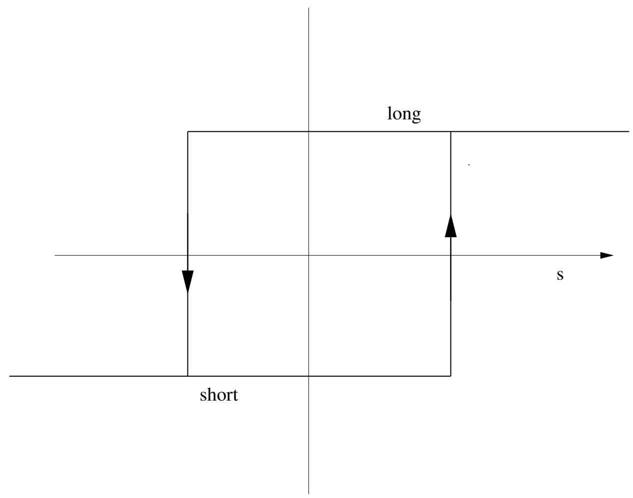

FIGURE 1. A hysteretic trader

recouped should a buy or sell decision be reversed. Such costs can be large relative to the more explicit transactions costs [18]. An intuitive interpretation of the trader thresholds is as the bid and ask limit order prices traders are willing to post in the limit order book.

Below we will use $+(-)$ to denote that a trader is long (short) and index the traders by $i \in 1, \ldots, N$. If trader i is long, we have

$$\frac{ds_i^+}{dt} = -\gamma_i^+ s_i^+ + I_i^+(t) \tag{1}$$

for as long as $s_i^+ \in (-1, \infty)$. If, at some time t_0, $s_i^+(t_0) = -1$ then trader i goes short and her propensity variable becomes $s_i^-(t_0) = -1$. The evolution then continues following the equation

$$\frac{ds_i^-}{dt} = -\gamma_i^- s_i^- + I_i^-(t), \tag{2}$$

for $t > t_0$ until $s_i^-(t) = 1$.

Here $I_i^+(t)$ is the information stream of the i-th trader if she takes a $+$ (long) position, etc. We shall assume that

$$I_i^+(t) = I_i^-(t) = I(t),$$

i.e. the information is identical for all traders and is state-independent. As discussed in Section 2, this is tantamount to assuming that all the external information streams are public in nature, which is more appropriate for limit order markets. We will discuss the information stream in more detail below. The $\gamma_i^\pm$ are the "inertia" coefficients of the i-th trader, and again, for simplicity we will assume here that

$$\gamma_i^\pm = \gamma,$$

so that, for example, two long traders are only differentiated by their s^+ values. This γ term can also be thought of as 'mean-reversion', ensuring that traders do not drift arbitrarily far from 0. This is reasonable if one assumes that information will be 'forgotten' or become less relevant as time passes; and that if the information stream is turned off, each agent's propensity for action will drift to 0 without switching.

We follow the exposition of [22] to specify the external information stream $I(t)$. For that, we need the densities of long and short traders, $\rho^+(s,t)$, $s \in (-1,\infty)$ and $\rho^-(r,t)$, $r \in (-\infty,1)$, the equations for whose evolution will be derived in the following sections.

We take

$$I(t) = \sum_k \epsilon^{(k)} \delta(t - t^{(k)}),$$

where δ is the Dirac delta, and $t^{(k)}$ is the time at which a piece of information with impact $\epsilon^{(k)}$ arrives. For simplicity, we allow only information impacts $\epsilon^+ > 0$ and $\epsilon^- < 0$; the case of $\epsilon^+ + \epsilon^- = 0$ is the case of symmetric information impact.

The last question to be settled is the arrival frequency of information. We will take this to be $\nu^\pm$ for positive (negative) information. Omurtag and Sirovich [22] suggest a simple way of incorporating "mimesis", i.e. interdependence of decision-making between the traders, which in our set-up leads to the following argument for $\nu^\pm$. Let us define the rate of going long for short traders to be

$$R^+ = \nu^+ \int_{1-\epsilon^+}^{1} \rho^-(x,t)\, dx. \tag{3}$$

Then if traders interpret the adoption of a long position by others as positive information,

$$\nu^+ = \nu_{ex}^+ + \alpha^+ R^+, \tag{4}$$

where ν_{ex}^+ is the exogenous positive information arrival frequency and α^+ is a measure of positive mimesis. From (3) and (4) we have that

$$R^+ = \frac{\nu_{ex}^+ \int_{1-\epsilon^+}^{1} \rho^-(x,t)\, dx}{1 - \alpha^+ \int_{1-\epsilon^+}^{1} \rho^-(x,t)\, dx} \tag{5}$$

and

$$\nu^+ = \frac{\nu_{ex}^+}{1 - \alpha^+ \int_{1-\epsilon^+}^{1} \rho^-(x,t)\, dx}.$$

Similarly,

$$\nu^- = \frac{\nu_{ex}^-}{1 - \alpha^- \int_{-1-\epsilon^-}^{-1} \rho^+(x,t)\, dx}.$$

Since $\rho^\pm \leq 1$, these quantities are well-defined at least if $\alpha^\pm < 1$; in our simulations much larger values of $\alpha^\pm$ (up to $\alpha^\pm \approx 200$) also lead to acceptable dynamics.

Clearly, the traders that we are considering are hysteretic in the sense that if we only know that the propensity for action of i-th trader is anywhere in the interval $(-1,1)$, we cannot deduce deduce from that the position of the trader in the market. A different context where the individual dynamics of (1)-(2) makes sense is a voting system in which a voter can only switch allegiance between two candidates.

4. **Mesoscopic equations.** By considering an interval $[x, x+\Delta x]$, $x \in (-1,\infty)$, and balancing the traders leaving and entering the interval due to drift towards the origin and jumps due to arrival of information, and then letting $\Delta \to 0$, we arrive at the following PDE for $\rho^+(x,t)$, $t > 0$.

$$\begin{aligned}(\rho^+)_t = {} & (\gamma x \rho^+)_x + \nu^+(\rho^+(x-\epsilon^+,t) - \rho^+(x,t)) + \nu^-(\rho^+(x-\epsilon^-,t) - \rho^+(x,t)) \\ & + R^+ \delta(x-1),\end{aligned} \tag{6}$$

for $x \in (-1, \infty)$. Similarly, for ρ^- we have

$$(\rho^-)_t = (\gamma x \rho^-)_x + \nu^+(\rho^-(x-\epsilon^+,t) - \rho^-(x,t)) + \nu^-(\rho^-(x-\epsilon^-,t) - \rho^-(x,t)) \\ +R^-\delta(x+1), \tag{7}$$

for $x \in (-\infty, 1)$.

Note that that equations are coupled through $\nu^\pm$, $R^\pm$ and are nonlinear and nonlocal.

4.1. **Fokker-Planck equations.** Expanding the jump terms in $\epsilon^\pm$, we obtain

$$(\rho^+)_t = (\mu\rho^+)_x + \frac{1}{2}\sigma^2(\rho^+)_{xx} + R^+\delta(x-1), \tag{8}$$

where

$$\mu = \gamma x - \nu^+\epsilon^+ - \nu^-\epsilon^-$$

and

$$\sigma^2 = \nu^+(\epsilon^+)^2 + \nu^-(\epsilon^-)^2 > 0.$$

Similarly, for ρ^- we have

$$(\rho^-)_t = (\mu\rho^-)_x + \frac{1}{2}\sigma^2(\rho^-)_{xx} + R^-\delta(x+1), \tag{9}$$

Since we require that

$$\frac{d}{dt}\left(\int_{-\infty}^{1} \rho^-(x,t)\,dx + \int_{-1}^{\infty} \rho^+(x,t)\,dx\right) = 0,$$

(Formally) integrating and using (8)–(9), we see that sufficient conditions for probability conservation are

$$\mu\rho^\pm(x,t) + \frac{1}{2}\sigma^2(\rho^\pm)_x \to 0 \text{ as } x \to \pm\infty, \tag{10}$$

while at the jump points

$$\begin{aligned} -\mu\rho^+(-1,t) + R^- - \frac{1}{2}\sigma^2\rho^+_x(-1,t) = 0, \\ \mu\rho^-(1,t) + R^+ + \frac{1}{2}\sigma^2\rho^-_x(1,t) = 0. \end{aligned} \tag{11}$$

(10)–(11) are the boundary conditions that we will be using below. Recall that μ, σ, $R^\pm$ are all functions of the densities, so these are non-local, nonlinear, time-dependent boundary conditions.

5. **Numerics.** Below we show some numerical simulations of the Fokker-Planck equations (8) and (9) using a fully implicit one-step finite difference method developed by Chang and Cooper [6]. The method is highly stable, computationally efficient due to the tri-diagonal structure of the finite difference matrix and, as noted in [23], preserves the non-negativity of the density functions under very weak conditions on the diffusion and advection coefficients.

In our simulations, the density ρ^+ is confined to the interval $[-1, 10]$ and ρ^- is confined to the interval $[-10, 1]$ with zero-flux boundary conditions at -10 and 10 to conserve probability. At each time-step the switching rates $R^\pm$ and the total information arrival rates $\nu^\pm$ are computed via (3) and (4). The masses $R^\pm\Delta t$, where Δt is the time-step, are moved between the positive and negative densities at $x = \pm 1$ and the boundary conditions (11) are enforced. In all simulations we used $\gamma = 0.2, \epsilon^+ = -\epsilon^- = 0.1$; these values were chosen to correspond roughly to the parameter regime studied in [22].

We start by considering the effect of the mimesis parameters $\alpha^\pm$. Figure 2 shows the two density functions at equilibrium with $\alpha^+ = \alpha^- = 0$ and $\nu^+ = \nu^- = 40$. If both $\alpha^\pm$ are kept equal and increased then there is very little difference in the equilibrium state until, as can be seen from the denominator of (5), eventually the system becomes both unphysical (as the densities stop being non-negative) and unstable with either very large and positive or negative values for $R^\pm$. Noticeable differences in the equilibrium state are observed for unequal $\alpha^\pm$ values, as can be seen in Figure 3 where $\alpha^+ = 50$ and α^- is kept at 0.

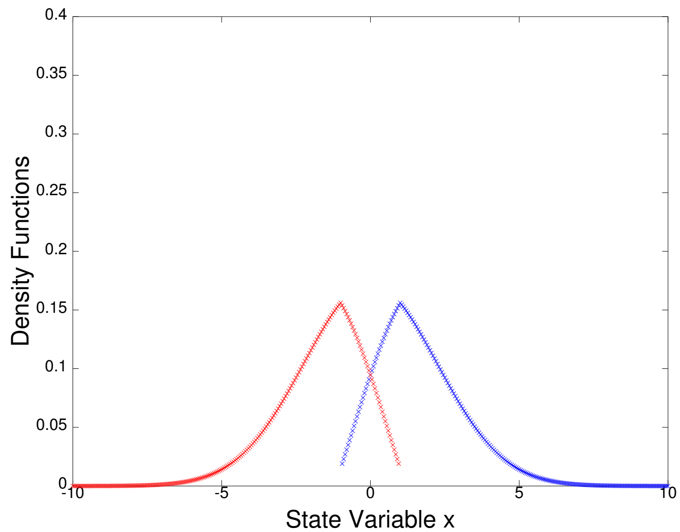

FIGURE 2. The equilibrium densities ρ^+ (on $[-1, 10]$) and ρ^- (on $[-10, 1]$) in the absence of mimesis.

We now turn from equilibrium solutions to the effects of varying the exogenous information rates $\nu^\pm$ over time. Figure 4 plots the total positive density $\int_{-1}^{10} \rho^+(x)\, dx$ against ν^+ which cycles adiabatically between 0 and 40 (whilst keeping the total information level $\nu^+ + \nu^- = 40$). The resulting loop clearly demonstrates the existence of hysteresis at the aggregate level.

Finally we consider the effect of randomness in the information streams. In Figure 5, at each time-step both ν^+ and ν^- are chosen from the distribution $200|\mathcal{N}(0, \Delta t)|$ where the time-step Δt is 0.05 and $\alpha^+ = \alpha^- = 10$ (these parameters are arbitrary but of the same magnitude as those used in the earlier simulations). Note that as an initial approximation we are assuming that the information frequencies are uncorrelated, both with each other and themselves.

The plot shows the total positive density $\displaystyle\int_{-1}^{10} \rho^+(x,t)\, dx$, which can be considered a proxy for price (or the excess of positive over negative sentiment) as a function of time. The system does not exhibit any tendency to converge to the mean value of $\frac{1}{2}$ with large deviations being the rule rather than the exception.

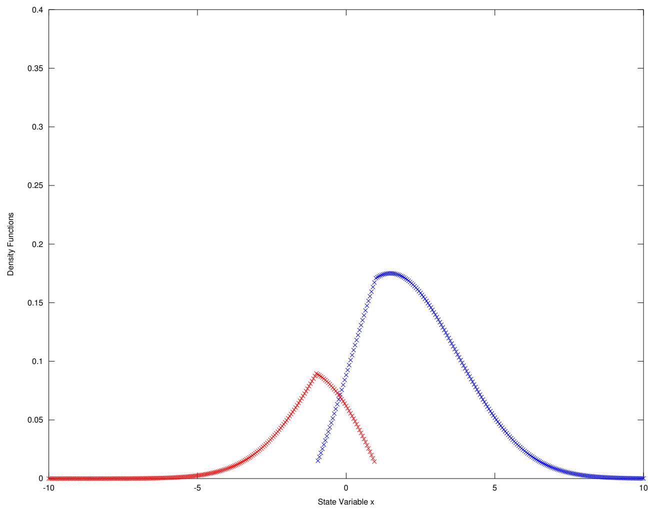

FIGURE 3. An asymmetric equilibrium solution where α^+ has been increased to 50.

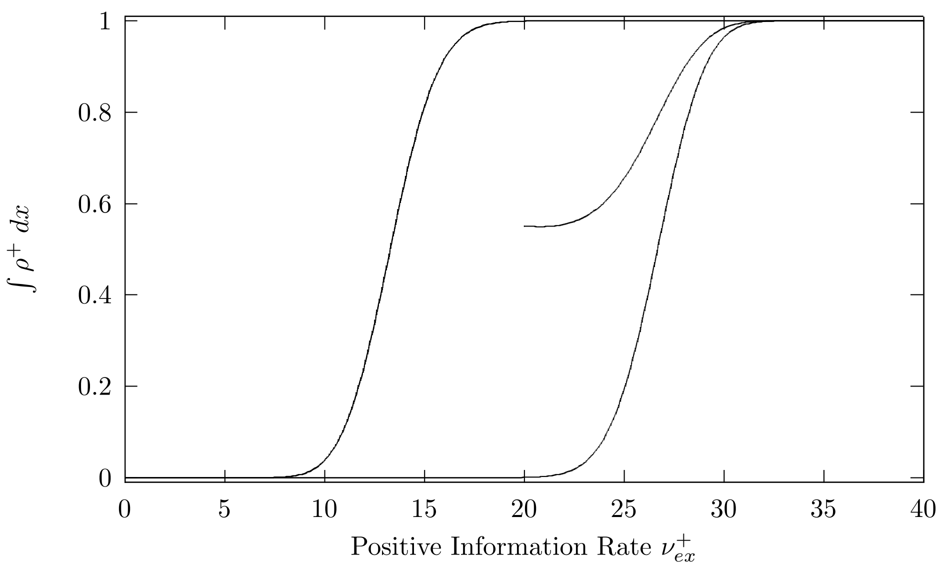

FIGURE 4. A hysteresis loop caused by adiabatic changes in the relative frequency of positive and negative information. The system is started from a neutral (randomized) state which results in an interior curve until saturation is reached for the first time.

This can be interpreted as bubbles in the market: in the present framework, a bubble corresponds to the concentration of trader density in one predominant market position without the fundamentals of the market (measured by $\nu_{ex}^{\pm}$) justifying it (e.g. if, as in Figure 5 the expectation of ν_{ex}^{+} equals that of ν_{ex}^{-}).

Furthermore, there appear to be periods of higher volatility associated with larger and more sudden changes in sentiment. This is more clearly seen in Figure 6 which plots the changes (first differences) in $\int_{-1}^{10} \rho^+(x,t)\,dx$ from one time-step to the next. This phenomenon is known as *volatility clustering* and is an extremely important property (or 'stylized fact') of almost all financial markets. Note that since the information frequencies are uncorrelated this effect is entirely due to endogenous dynamics.

The dashed curve in Figure 7 plots the autocorrelations of the data from Figure 6 for time-lags from 1 to 50 timesteps. As is the case with price change data for real markets, this autocorrelation very rapidly vanishes to zero. However the solid curve shows the autocorrelations of the *magnitudes* of the same data (which is a commonly-used definition of price volatility) and this decays to zero only very slowly — a quantifiable indicator of the volatility clustering that is present. [1]

Finally we consider the other most significant stylized fact, *excess kurtosis* or 'fat-tails'. Figure 8 (solid line) shows a cumulative plot of the magnitudes of the data from Figure 6 on a log-log scale. The dashed line shows the same plot for a Gaussian distribution with the same distribution as the underlying data. Gaussian distributions have exponentially decaying tails while financial data (and many other natural and socio-economic phenomena) decay far more slowly, often with observed approximate power-laws. This is highly significant for risk models that can underestimate the likelihood of extreme market moves by many orders of magnitude. Figure 8 displays clear evidence of excess weight in the tail and even an approximate power-law decay as indicated by the near-linear decay.

6. **Concluding Remarks.** In view of the above numerical simulations, the equations (8)-(9) are worthy of further study. Global existence in the presence of herding seems a non-trivial issue. For example, it is not clear how to derive a bound for the maximal allowable values of $\alpha^{\pm}$. We also leave proof of stabilisation and dependence of bubble-formation and volatility clustering on the statistics of $\nu_{ex}^{\pm}$ and $\alpha^{\pm}$ to future work. On the other hand, the uniqueness of (continuous in space) steady states can be easily verified by explicit but cumbersome construction.

The Fokker-Planck type equations proposed here can be used to model systems with larger number of states, and Preisach-type systems as well. Briefly, for the latter, if the probability that a short trader goes long at $x = u$ is not a Dirac delta at $u = 1$, but is given by a distribution function $g^+(u)$ with support in a neighbourhood u^+ of 1 in $(-\infty, 1]$, $u^+ \cap 0 = \emptyset$, then if we set

$$r^+(u) = \nu^+ g^+(u) \int_{u-\epsilon^+}^{u} \rho^-(x,t)\,dx,$$

[1] What this means for actual markets is that a day on which there is a large price change (in either direction) is more likely to be followed by another large-price-change day but it is just as likely to be a price change in the opposite direction as the same direction.

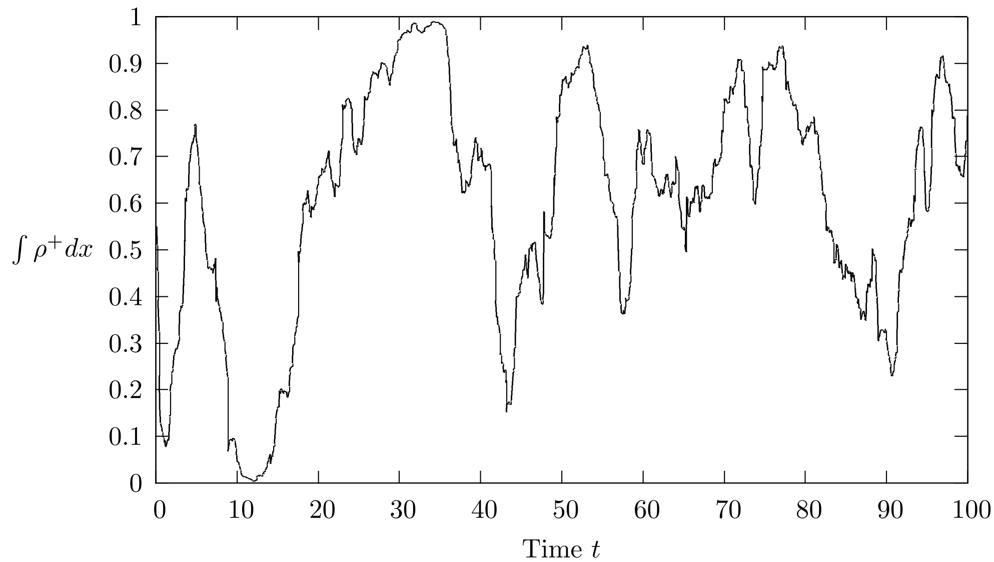

FIGURE 5. A plot of $\int_{-1}^{10} \rho^+(x,t)\,dx$ against time. Even though the expected values of ν^+ and ν^- are the same, the system does not converge to a steady state with equal numbers of agents in each state.

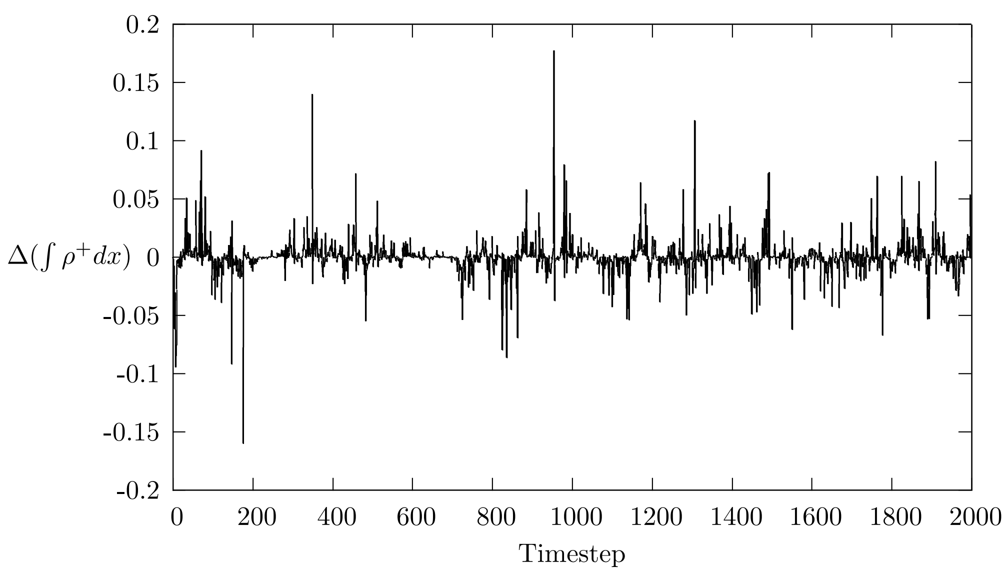

FIGURE 6. A plot of the first differences from Figure 5. The existence of periods of higher volatility is a near-universal feature of financial markets and is known as volatility clustering.

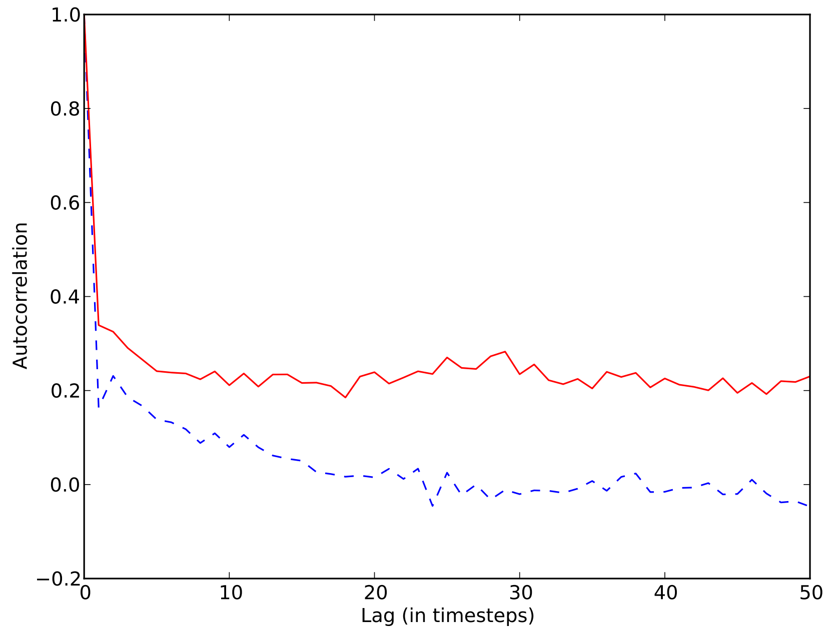

FIGURE 7. The linear autocorrelation of the data plotted in Figure 6 is shown as the dashed line and rapidly falls to zero. However, the autocorrelation of the absolute values (solid line) decays very slowly.

and $r^+ = \int_{u^+} r^+(u)\,du$, then, as before, $\nu^+ = \nu^+_{ex} + \alpha^+ R^+$ and the Fokker-Planck type equation for ρ^+ is

$$\rho^+_t = (\mu\rho^+)_x + \frac{1}{2}\sigma^2(\rho^+)_{xx} + r^+(u)\delta(x-u),$$

with the equation for ρ^- being derived using similar logic. However, unlike in classical Preisach models, the resulting process is not expected to be rate-independent.

We note that in principle more realistic dynamics of individual traders can be incorporated into (1)–(2) which will be reflected in the diffusion and advection coefficients of the resulting Fokker–Planck equation and in the coupling terms. From the economics point of view, it would be helpful to incorporate price dynamics into the equation, which would pave the way for deriving a mesoscopic version of [11].

At the Alexei Pokrovskii Memorial Conference it was suggested that each trader might respond to a mixture of public and private information reflecting the presence of dealers operating alongside the limit order book in hybrid stock markets. While such a mixed information stream is easily enough incorporated in a heterogeneous agent model, this would not be straightforward in a mesoscopic model.

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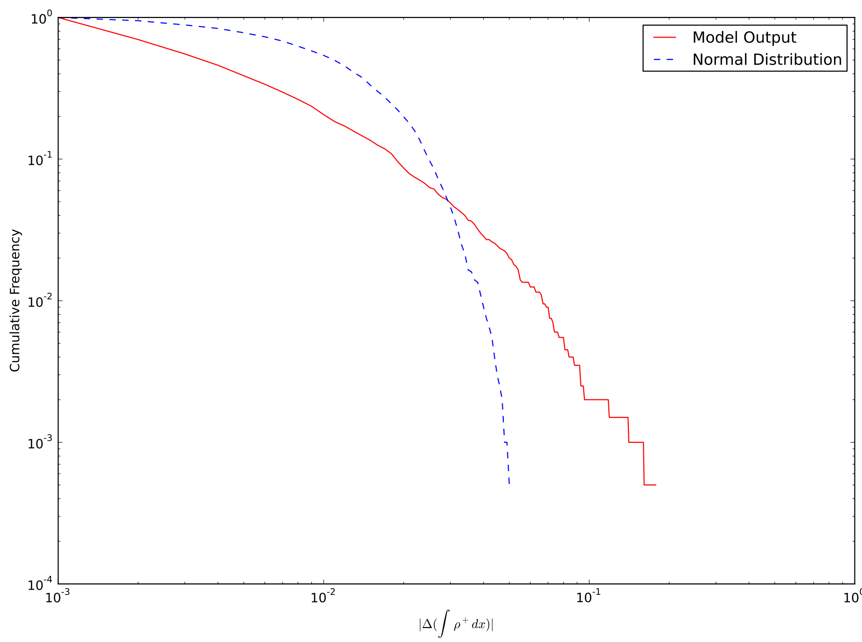

FIGURE 8. A cumulative log-log plot of the absolute values plotted in Figure 6 (solid line). For comparison the dashed line shows data from the Normal distribution with the same variance. There is clear evidence of a fat-tailed distribution and excess (lepto-) kurtosis.

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