# Hysteresis and Economics

## Taking the economic past into account

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In this article the thinking behind the applications of notions of hysteresis to economic systems is discussed. In particular, it is explained why the idea that many aspects of economic systems are hysteretic is a plausible and attractive one. The attempt is to be as explicit as possible about the difficulties encountered when trying to incorporate hysteretic effects into models that can be validated and perhaps used as tools for macroeconomic control. The growing understanding of the ways in which memory effects influence the functioning of economic systems is a significant advance in economic thought and, by removing distortions that result from oversimplifying specifications of input–output relations in economics, this way of thinking has the potential to narrow the gap between economic modeling and economic reality.

It is useful to begin by explaining in broad terms why the framework of hysteresis is appropriate in economics. There are two reasons, one relatively old and vague, the other more recent and precise.

It is trivially true that economic systems are historical formations since they are the results of evolutionary processes, though not only of evolutionary economic processes. Recent East European history shows that trying to set up an economic system *ab initio* is a fraught undertaking. A clear understanding of the rootedness of economic processes can be found in [1], which is also one of the first works in economics to mention hysteresis explicitly.

The intuition that a deep insight into the forces driving economic processes necessitates taking into account the unfolding of economic structures in time is certainly not enough to single out any particular mechanism of history dependence at work in economic systems. The other observation that points towards hysteresis, and in particular to Preisach–type models, is the realization that economic agents are heterogeneous with respect to their preferences, propensity for action, and ranges of economic action available to them, and that in many cases of interest an economic agent can be represented by a *hysteron*. A hysteron is a simple input–output system with weak, passive hysteresis, as defined below. Sometimes though, as in models of stock market dynamics, an economic agent is better represented by a more complex input–output system with hysteresis.

The extent of the usefulness of hysteretic thinking in economics depends on the faithfulness of the representation of economic agents as constituting systems with hysteresis. Such a viewpoint makes it possible to understand the provenance of hysteresis loops in relations between macroeconomic variables and to introduce into the analysis of diverse spheres of economic activity the concepts thresholds, rate independence, and remanence [2]. The key fact that argues for hysteretic behavior at the micro-level is the existence of sunk costs associated with switching between strategies.

The structure of this article is as follows. First static hysteresis input–output (I/O) systems, hysterons, and Preisach-type models are defined, and the form that macroeconomic models with hysteresis typically take is described. Then the economics background needed by non-economists is sketched, and the distinctive nature of models in economics is pointed out. In the following central section of the article the results of approximately two decades of hysteresis modeling in economics are summarized and critically assessed. In that section we also describe in detail the thinking behind considering economic agents as input-output systems with hysteresis. Finally, stray thoughts are collected and pointers to the future in this area of research are highlighted.

## Systems with hysteresis

In this section the models described in subsequent sections are located within the general theory of systems with hysteresis. This theory has matured significantly in the last 20 years; for details on its physical roots, application, and mathematical structures the reader is referred to [3], [4], [5], [6], [7] and the economics–oriented exposition in [8].

Consider a system with a scalar input $u(t)$, an output $s(t)$, and some initial state $s(0) = s_0$. This I/O system is a *system with memory* if, at time $t$, the output is determined by the history of inputs, the set $\{u(\tau),\ \tau \in (0,t]\} \subset \mathbb{R}$, and not just by $u(t)$. Therefore there is no single-valued mapping from $\mathbb{R}$ into $\mathbb{R}$ that associates all possible values of output $s(t)$ with an input $u(t)$; instead, such a mapping, which we will denote by $\mathcal{H}$, is set-valued. In other words, $\mathcal{H} : \mathbb{R} \to 2^{\mathbb{R}}$, and for all $t$, $s(t) \in \mathcal{H}(u(t))$. Denote the I/O relationship in a system with memory by

$$s(t) = F(\{u(\tau),\ \tau \in (0,t]\}) := F[u](t).$$

Note that the this relationship defines a mapping between function spaces. Fix some time $T > 0$ and let $u$ be an element of a space $X$ of real-valued functions defined on $[0,T]$, for example $X = C([0,T])$, or $X = L^p([0,T])$. Define a function $s$ on $[0,T]$ pointwise by $s(t) = F[u](t)$. The mathematical analysis of systems with memory requires knowing what space $Y$ $s$ belongs to, and whether the mapping $\mathcal{F} : X \to Y$, $\mathcal{F}(u) = s$ is continuous.

Suppose now that time is reparameterized by a mapping $t \mapsto h(t)$, where $h'(t) > 0$, and set $u_h(t) = u(h(t))$. If, for every reparameterization $h$ and every value of $t$, it follows that $F[u_h](t) = F[u_h](t)$, then the system is *rate independent*. For applications in economics the most appropriate definition of a hysteretic I/O system is the following one.

**Definition:** A *hysteretic* I/O system is a rate independent I/O system with memory.

The quintessential example of a hysteretic I/O system is a *hysteron* [3] (this object is called a relay in [5]). It is an I/O system which is defined as follows. Let $\alpha, \beta$ be real numbers with $\alpha < \beta$ and assume that there exists a time $t^*$,

$$t^* = \max\{\tau \in (0, t] \,|\, u(\tau) = \alpha \text{ or } \beta\}.$$

For a hysteron, the output $s(t)$ is given by

$$s(t) := F_{\alpha\beta}[u](t) = \begin{cases} 1, & \text{if } u(t) \geq \beta \text{ or if } u(t) \in (\alpha,\, \beta) \text{ and } u(t^*) = \beta, \\ \\ -1, & \text{if } u(t) \leq \alpha, \text{ or if } u(t) \in (\alpha,\, \beta \text{ and } u(t^*) = \alpha. \end{cases}$$

The induced mapping $\mathcal{F}_{\alpha\beta} : X \to Y$ is not a continuous mapping for any choice of function spaces $X$ and $Y$. Note also that the output $s(t)$ is constrained to lie on the union of two curves in $\mathbb{R}^2$, $C_L = \{(x, -1),\, x \in (-\infty, \beta)\}$ and $C_U = \{(x, 1),\, x \in (\alpha, \infty)\}$. This observation motivates the following definition (see [5]).

**Definition:** If in an I/O system with hysteresis the set-valued map $\mathcal{H}$ has the property that for all $u \in \mathbb{R}$, the set $\mathcal{H}(u)$ contains a finite number of points, then the I/O system has *weak hysteresis*. Otherwise it has *strong* hysteresis.

Consider a system with hysteresis at time $t_0$ with output $s_0 = s(t_0)$ and input $u_0 = u(t_0)$.

In the absence of changes in the input, the output remains the same. Now suppose that the input $u(t)$ changes from $u_0$ to some value $u_1$ and back again. Then for each $u_0$ there are values $u_1$ such that, after the excursion, the output does not return to $s_0$ but instead to some different value $s_1$. For a hysteron in state $s(t_0) = -1$ and $u_0 \in (\alpha, \beta)$, examples of such values would be $u_1 \geq \beta$ or $u_1 \leq \alpha$. This is the phenomenon of *remanence*. To return the output variable to its original value $s_0$, we need to change the input by an additional amount, commonly called the *coercive force*. These concepts are illustrated in Figure 1.

The counterclockwise dynamics of a hysteron under a periodic input is typical of systems with weak hysteresis (counterclockwise traversal of a hysteresis loop is often referred to as *passive* hysteresis [9]). However, the dynamics of hysteretic elements can be much more complex; towards the end of this article an example of a model of stock-market dynamics is provided, following [10], where an element flips state whenever either threshold value is passed and the thresholds themselves change each time the state changes. Nevertheless such an element is hysteretic by our definition and displays remanence. Note that in the hysteron described above, if threshold values are crossed, we only need to know the historical record of the input to be able to predict the state of the element; in the model of [10] the initial condition continues to influence the dynamics no matter what the input later does.

In economics applications of hysteresis it is often natural to assume that the input $u(t)$ evolves in discrete time $t_k \in \mathbb{Z}$. Thus, if $u(t_k)$, $s(t_k)$ are given and $u(t_{k+1})$ is prescribed next, we assume that $u(t_k)$ and $u(t_{k+1})$ are interpolated by any monotone continuous function $\widehat{u}(t)$, where $t \in [t_k,\, t_{k+1})$, $\widehat{u}(t_k) = u(t_k)$, $\widehat{u}(t_{k+1}) = u(t_{k+1})$, and $s(t) = F[\widehat{u}](t)$, $t \in [t_k,\, t_{k+1})$.

Many economic systems can be represented by large assemblages of heterogeneous elements, each having simple dynamics, for example, that of a hysteron. However, it is not clear whether an assemblage of hysteretic elements under a common input $u(t)$ is itself a hysteretic I/O system with some appropriately defined aggregate output variable. A useful class of strong (passive) hysteresis I/O systems arises when we set

$$s(t) = \mathcal{P}[u](t) := \int_{\Gamma} g(\alpha, \beta) F_{\alpha,\beta}[u](t) \, d\alpha \, d\beta,$$

where $g(\alpha, \beta)$ is a continuous function with support in $\Gamma \subset \{(\alpha,\, \beta) \in \mathbb{R}^2 \,|\, \beta \geq \alpha\}$, such that

$$\int_{\Gamma} g(\alpha, \beta) d\alpha \, d\beta = 1,$$

and $|\Gamma| \neq 0$. Such systems are called ***Preisach-type*** systems. The function $g(\alpha, \beta)$ is the ***Preisach weight (density) function***. The standard reference for the theory, applications, and generalizations of such models is [11]. Preisach hysteresis operators $\mathcal{P}[u]$ have good continuity and monotonicity properties [6], [5], which makes systems with Preisach hysteresis especially amenable to analysis. The dynamic features of Preisach-type systems, such as the remanence property discussed above, as well as the *wiping–out*, and the *congruence* properties, are described by the *Mayergoyz staircase construction* [12], [11], [8]. In brief, an I/O system has the wiping–out property if the output is uniquely determined by the sequence of non-dominated optima of the input; and a system with strong hysteresis has the congruence property if all the interior loops generated by cyclical excursions of input between two values, say $u_0$ and $u_1$, $u_0 < u_1$, are geometrically congruent (see, for example, [12, Figure 2]). Systems with Preisach hysteresis operators in discrete time have been considered, for example, in [13].

A Preisach-type I/O system is an example of a system that exhibits hysteresis at both the micro and the macro level. However, many models that give rise to macroscopic hysteretic I/O systems do not require hysteretic units at the micro level. Perhaps the best-known class of such models is the random field Ising model of Barkhausen noise [14], [15] (for a recent economics application see [16]). The key to obtaining hysteresis at the macroscopic level is to assume enough heterogeneity among the units (in the form of thresholds chosen at random from some distribution) and to couple them appropriately. More precisely, if the system consists of $N$ units with the $i$-th unit at time $t_k$ having states $s^i(t_k)$, say with $s^i(t_k) \in \{0, 1\}$, the zero-temperature random field Ising dynamics [14] update rule is given by

$$s^i(t_{k+1}) = \text{sgn}\left[\sum_{j=1}^{N} J_{ij} s^j(t_k) + f^i + u(t_k)\right],$$

where $f^i$ are the random thresholds and $J_{ij}$ are the unit-coupling parameters. Note that there is no coupling between elements in a Preisach-type system (as this would destroy the distinction between input and output) but assuming that the elements are decoupled is too restrictive in economics. Hence an appropriate class of models in economics is one in which elements are hysteretic, time is discrete, and elements are coupled. Symbolically, such a system can be written (if we assume a Preisach-type hysteresis operator), as

$$\begin{aligned} u(t_{k+1}) &= u(t_k, s(t_k), \ldots) \\ s(t_{k+1}) &= \mathcal{P}[u](t_{k+1}), \end{aligned} \tag{1}$$

where ellipses may stand, for example, for exogenous shocks. A model of this type has been used in [17] to describe market entry-exit decisions of firms.

## Economics background

The importance of incorporating hysteresis into the analysis of economic systems can be seen by considering the limitations of mainstream economic analysis, which excludes memory effects.

The foundations of the mainstream model of contemporary economics were laid in the "neoclassical revolution" of the 1870s. In contrast to the preceding "classical economics", where the analysis focused on capitalists, workers and landlords considered as collective entities, the starting point in neoclassical economics was the individual economic agent. Households consume goods and accumulate assets through savings and supply labor inputs into production. Firms produce goods, use savings to finance production, and have a demand for the labor and capital inputs used in production. These economic agents are characterized as being predominantly self-interested. Households maximize utility functions that depend upon the goods they consume and the labor services they supply, subject to budget constraints matching expenditure and income. Firms maximize their profits, determined by revenues minus costs, subject to production function constraints describing feasible production techniques. From this postulated behavior, individual demand and supply schedules are derived, which depend on the prices of the consumer goods and production inputs. In the simplest set up, households and firms are price takers, with the agents regarding the prices as given outside of their control. Individual demand and supply schedules are then aggregated to form market demand and supply functions, the functional dependence being on the prices.

The key question is then whether there is a set of prices that can reconcile the interests of buyers and sellers so that aggregate quantities demanded equal aggregate quantities supplied on all markets. Initially this question was addressed by invoking a *deus ex machina*, in the form of an unpaid auctioneer, who would announce a set of prices, record the excesses of demand over supply, and converge to a market-clearing price vector by adjusting prices upward (downward) more in markets with the greater excess demand (supply). This unsatisfactory solution, proposed by the engineer-trained Walras in 1874 [18], relied on the number of unknown prices being at least matched by the number of equations in the form of excess demand functions. Modern Arrow–Debreu proofs of the existence of such a market-clearing price vector use fixed point theorems and rely on assumptions such as convexity in consumption and production sets [19].

The original pioneers of neoclassical economics tended to have a mathematics, physics, or engineering background and relied heavily on metaphors drawn from Newtonian mechanics when constructing their theory of value, that is the determination of relative prices [20]. Market equilibrium was thus seen as a balance of forces, responses of quantities to changes in prices were defined in terms of elasticities, and in Fisher's account the individual agent was seen as a particle, the commodity was seen as a type of space, marginal utility corresponded to force, disutility to work, and utility to energy [21, p. 85]. Indeed a central place in Fisher's Ph.D. thesis is taken by a hydrostatic model of water flowing through pipes to interconnected cisterns. This model illustrates how the marginal utility of consumption and the marginal cost of production are brought into balance at the market equilibrium price. The key characteristic of this theory of value is that equilibrium market prices reflect the marginal utility of the marginal consumer and the marginal cost of the marginal producer. Figure 2 reproduces Fisher's diagrammatic representation of his model [21, p. 56].

From the 1930s neoclassical economics was reformulated on an axiomatic basis, but the mathematical techniques used in the general equilibrium existence and stability proofs preserved the properties imported by the original metaphors of Newtonian mechanics [20], [19]. In particular, conservation of energy was retained in the economic context to imply that nothing is lost or permanently changed if an individual, a market, or the economy as a whole face a temporary disturbance, no matter how large. For example, during the US Civil War, the burning of the cotton fields reduced the supply of cotton. Adherents of the views described above would have predicted that the market for cotton would return to the *status quo ante* after the war. The reduction of cotton supply could not, in their opinion, have lasting effects. They expected the replanting of the cotton fields *post bellum* to lead to a return to the original equilibrium quantity and price, with utility functions describing tastes and production functions describing production possibilities being unchanged. Similarly, proponents of such views expected the US recession of 1929–31 to have merely temporary effects, economic output returning to its prior trend rate of growth during the subsequent recovery phase. The processes involved are seen as reversible, time symmetry being preserved as seen in relative prices returning to their level before the disturbance.

The limitations of this economic framework, which does not allow lock-in due to irreversible changes, were understood already by Marshall, one of the great expositors of neoclassical economics [22, pp. 425–426, p. 660, p. 667]. The incorporation of hysteresis into the analysis of economic systems addresses at least some of Marshall's criticisms.

**Macroeconomic terms**

The main concerns of *macroeconomics* are with the determination of aggregate output, unemployment rate, and inflation. We now describe briefly the standard account of the dynamics of these indicators of economic activity.

*Aggregate output* is measured in national accounts using definitions such as gross domestic product (GDP). Relations link instantaneous national output, income and expenditure (once adjustments are made for international trade, government expenditure, taxation, and so on). Say's Law, that "supply creates its own demand", summarizes the neoclassical wisdom that there cannot be a shortage of demand provided that the forces underlying supply and demand are not fettered by government interference. The value added in production is distributed as income to the factors of production. Income is spent on consumption goods or saved. Savings would be channeled into satisfying the investment demand for new capital goods, the rate of interest moving to eliminate any discrepancy between the demand for and supply of loanable funds. On this view, business cycle downswings are associated with disturbances such as bad harvests — caused by sunspot activity variations in one account — but would be temporary, the losses in output being recovered in the subsequent upswing phase.

The term *unemployment* only came into usage in the English language in the 1880s. Any lack of employment tended originally to be attributed to physical or moral deficiencies in those without work and not to coordination failures in economic systems. In present day so-called *natural rate of unemployment* theories of neoclassical economics [23], the equilibrium or "natural" rate of unemployment that is consistent with a steady rate of inflation, depends largely on the degree of government intervention in the form, *inter alia*, of minimum wages, state unemployment benefits and trades union or labor market regulations. Such interventions raise real wages above the market-clearing levels. Thus, according to this natural rate theory, if unemployment is perceived to be too high, the solution is to dismantle government interventions in labor and other markets.

*Inflation* is measured as the rate of change of some index of prices, the main headline figure being that for consumer prices. The traditional quantity theory of money explains the rate of inflation by the prior change in the quantity of money. In the early 20th century the quantity theory was formalized as $MV = PT$ by Fisher, where $M$ is the nominal stock of money, $V$ is the velocity of its circulation, $P$ is the level of the price index and $T$ is the flow of real transactions or the aggregate output flow. The velocity, $V$, was taken to be fixed by institutional arrangements such as cheque clearing, and $T$ by the conditions of aggregate supply. Hence the dichotomy between the theory of value and that of the price level. The causation was taken to be $M \rightarrow P$, ignoring the obvious problem of reverse causation. For more information on central bank attempts to control inflation, see “Inflation and Taylor rules”.

It was Keynes who presented the most influential challenge to the orthodoxy of neoclassical economics. He answered his question “Is the Economic System Self-Adjusting?” in the negative [24], arguing that “free” markets would not necessarily generate “ full employment”, which would be the “natural rate of (un)employment” in present-day terminology. The problem was that private-sector consumption and investment plans were formed in the face of expectations about the future economic environment that were inherently uncertain and subject to shifts that were more to do with emotions than with the rational calculations postulated in neoclassical economics. As a result, shortfalls of effective demand in relation to the level required for full employment occur. Hence potentially governments had a role to play by increasing their spending, cutting taxes or reducing interest rates to make up for any deficiencies in private sector demand. The Keynesian revolution stimulated the work of Phillips [25]; see “Phillips and his Machine” for details.

If a market-based economic system is not necessarily one that self-adjusts to “full” employment, as Keynes argued, there is a control problem. Tinbergen, the first director of the Dutch Central Planning Bureau in 1945, was a pioneer (see [26] for an account of this control literature in economics) and posed the control problem in the context of a set of fixed reference values for macroeconomic targets such as output, employment and the balance of payments. Econometric estimates of macroeconomic relationships were used to describe the impact of policy input variables, such as government spending and interest rates, on the output variables, a key issue being whether there would be a sufficient number of policy instruments to allow the simultaneous achievement of the reference values of the policy targets. There are many problems with the Tinbergen approach to macroeconomic control, including imperfections in the data describing the current and even past states of the economy, uncertainty about the best econometric description of the economy, uncertainty about the values of policy instruments set by foreign policymakers, and arbitrariness in the choice of reference target values. Subsequent approaches attempted to deal with these problems by framing the control problem in terms of a policy maker maximising a preference function defined over policy objectives, the national equivalent of an individual agent's utility function, subject to constraints describing the way the economy is perceived to function.

This approach to macroeconomic control has been out of fashion since the 1970s. The policy ineffectiveness critique associated with the natural rate hypothesis [23] claims that macroeconomic policy instruments or input controls do not have lasting effects on real policy targets such as unemployment and real GDP. Macroeconomic policies are claimed to have lasting effects only on nominal variables such as the rate of inflation, hence the switch of attention to how inflation policy targets can be achieved by Taylor-type rules [27]. The associated Lucas critique [28] argues that private sector agents have "rational expectations" in that they take into account all the relevant information contained in a model of their behavior when forming expectations. Thus their behavior will not be invariant with regard to the policy interventions of governments or their agencies. A related tendency has been to argue that the preference function of the policymaker should somehow reflect the preferences of individual economic agents, with policy outcomes reflecting a dynamic game played between policymakers and individual economic agents. The incorporation of hysteresis into the analysis of economic systems can not only provide new analytical foundations for Keynes' views on the existence of equilibria at less than full employment, but also resurrect the control problem denied by the policy ineffectiveness proposition of neoclassical economics.

## The methodology of macroeconomic modelling

As noted above, the mainstream model of neoclassical economics relies heavily on metaphors drawn from classical physics, Newtonian mechanics in particular, and in doing so imported conservation and reversibility principles into its analysis of economic systems. The obvious question is whether these properties are observed in the actual workings of economic systems. Alternative approaches to economic analysis tend to rely on metaphors drawn from contexts in which conservation and reversibility do not hold. For example, evolutionary economics takes evolutionary biology as its metaphor source, while the present article considers hysteresis, a term originally coined by Ewing to describe the behavior of electromagnetic fields in ferric metals, as its source [29]. This raises methodological issues as to how economists go about distinguishing the wheat from the chaff in their theories. Are evidential criteria paramount or does the evidence not play a decisive role? Can economists experiment, or are they condemned to draw fuzzy inferences from non-experimental data? (For more details on this, see "Experimental Economics".) Are there models that can be validated and used for prediction and control, or are models mere tools of thought used to elaborate how axiomatic principles might apply in particular circumstances?

Some of the changes that occur in actual economic systems can be considered as "natural" experiments. So researchers can look to see whether economies recover to their previous trend growth paths for output after a recession, or merely return to the actual previous level [30]. Or researchers can ask whether the move to inflation targeting by independent central banks has been accompanied by a reduction in the size of real GDP or unemployment fluctuations. In both examples the Duhem-Quine thesis again raises problems. Do the results obtained say yes or no to the hypothesis under test, or do they leave doubts as to whether other factors at work, such as institutional changes or globalization, might explain the findings?

The main vehicle for testing hypotheses in economics has been econometrics, which is concerned with drawing inferences from non-experimental time series, cross-section and panel data. Data on variables deemed to be exogenous or endogenous are used to attempt to identify structural economic relationships, or conduct tests on the nature of the reduced form relationships. A basic problem is that the error terms can reflect a wide set of phenomena such as measurement errors in the variables, excluded variables and model mis-specification as well as any inherent noise or non-stationarity in the underlying processes. Despite the waves of optimism that have accompanied the unveiling of new econometric techniques, the results have not yielded what might be described as highly-robust empirical findings. Some hypotheses can be regarded as unlikely because they are data-incoherent, but there are doubts as to the extent to which the apparently data-coherent relationships published in learned journals reflect the proficiency of the researcher in "data mining" rather than something more fundamental.

The problems involved in testing theories in economics help to explain why many economists appear to put a large weight on consistency with underlying principles, such as maximising behavior by economic agents, when choosing between rival theories. Evidence does play a part, but more in the way of qualitative properties such as whether economic processes display mean reversion or follow normal distributions. The Nobel laureate Samuelson, described his working methodology as an attempt to follow the injunction of Mach to provide an economical description/explanation of such "facts" [31].

On at least two key counts the hysteresis account of economic systems is promising. Firstly, neoclassical economics tends to simplify or ignore the aggregation problem by assuming the existence of "representative" economic agents, whose optimising decisions are scaled up to represent the behavior of all consumers or producers. Thus this theory is inconsistent with the observation that individuals differ and cannot capture interesting implications arising from agent heterogeneity (see [32] for a corrosive account). The ability of Preisach-type models to respect agent heterogeneity in their hysteron representations is certainly an improvement in this respect.

The second point relates to the issue of where consumer preferences or production techniques come from. In the neoclassical account, tastes and technologies are taken as exogenous to the economic system. This approach ignores the obvious presence of learning-by-doing in consumption and production.

## Hysteresis models in economics and finance

The rôle of hysteresis in economic and financial modeling is considered now. Most such models are based on representing individual economic agents as hysterons, an approach which provides, as is argued below, an attractive characterization of various micro-economic scenarios such as the entry and exit of firms in a particular market. The hysterons can then be aggregated to provide a Preisach-type macroeconomic model of total output and employment. As outlined earlier, much of neoclassical economics is predicated upon the existence of a unique, stable, history-independent equilibrium. Central banks in many countries, but interestingly not in the US, are restricted to using monetary policy to achieve a target rate of inflation. Yet remanence and coercivity in economic activity at the macroeconomic level are observed **REFS!!!** and such real effects of monetary policy have to be taken into account. Theoretically they can be accounted for by Preisach-type models of economic activity. The contrast between the mainstream model in finance, that of *efficient markets*, and models where hysteresis effects are present, is also considered.

**Economics**

First of all, the question needs to be asked whether and when can an economic agent be adequately represented by a hysteron, as previously defined. Such a representation is predicated on, firstly, there being a binary choice on the part of the agent, with the associated switching often involving what are termed *sunk costs*. The magnitudes of these sunk costs are a major factor in the determination of the threshold values $\alpha < \beta$ for a given agent and the ensuing heterogeneity of the model population. Also, the switching time of the hysteron must be fast compared to the time-scale of the model and the variations in the input $u(t)$. If at some point the input stops changing, the current states of the agents are maintained for a significant time and minimal switching occurs.

An example where these assumptions hold is a the following. Consider a simple case where the relative price of capital in terms of output is normalized to unity, so that one unit of capital is used to produce one unit of output. In the standard neoclassical account of investment decisions each firm estimates an internal rate of return (IRR) on each possible investment project. The IRR is the rate of return that would set the discounted value of future revenues net of operating costs equal to the projects capital costs. The IRR is then compared to the cost of capital funds, which can be written as markup $\lambda(t) > 1$ on the short term interest rate $i(t)$ set by the central bank, the repo rate at which the central bank lends money to private banks by repurchasing qualifying assets (see eq. (1)).

If investment projects were costlessly reversible, in that the capital costs of a project could be fully recouped should the project be abandoned, the *knife-edge" conditions* $\mathrm{IRR} > \lambda(t)i(t)$ or $\mathrm{IRR} < \lambda(t)i(t)$ would determine whether or not the firm would go or not go ahead with the project in the first place, or continue with or abandon the project if the latter is already under way. The evidence, however, is that firms require rates of return substantially in excess of the cost of capital funds, typically three or four times the cost of capital [33], before they proceed with investment projects. A highly plausible explanation for this is that capital projects involve sunk costs that are not recoverable should be the project be abandoned. If an oil exploration project is abandoned in the face of a fall in the price of oil, the second-hand price of the drilling rig is likely to be at a substantial discount to the purchase price; the sales, distribution network and advertising costs of bringing a new product to the market would be lost should the product flop; and so on. This account provides the rationale for reformulating the condition for the capital project to proceed as $[\mathrm{IRR} - \lambda(t)i(t)] \geq \beta$ where $\beta$ is the upper trigger in a Preisach model. Once a capital project has been begun the decision of the firm is whether or not to keep the project active. Because of the sunk costs, and also because of economic uncertainty regarding future net revenues — which depend *inter alia* on the expected future price of oil in the drilling rig example, and on fashion in the case of a new product — the firm will not abandon the project until a significantly lower trigger $[\mathrm{IRR} - \lambda(t)i(t)] \leq \alpha$ is reached.

Similar microeconomic foundations for the representation of economic agents as hysterons in Preisach-type models have been provided for the cases of how exports and imports respond to exchange-rate changes [8], how the hiring and firing of workers is related to shocks to aggregate demand, and how output in the member countries of the European Monetary union responds to the interest rates set by the European Central Bank (see [2] for a survey). The crucial element describing how economic agents respond to input variables is the presence of sunk costs in the adjustment of economic behavior. This conclusion matches the observation that in many contexts economic adjustments are made relatively infrequently, and in large doses, rather than responding more or less continuously to even small changes in input variable, as neoclassical models imply.

The qualitative properties of such Preisach-type economic models have been analysed using the general results on systems with hysteresis obtained in [3], with the staircase partition representation of the division between active and inactive hysterons/economic agents provided in [11] playing a prominent role. These models plausibly suggest that economic systems contain a selective, erasable memory of the non-dominated extrema of perturbations to input variables. In terms of business cycles this means that major recessions and booms leave permanent effects in their wake, rather than merely representing temporary deviations from some given growth path, as in the neoclassical account. The implications of Preisach-type models for business cycles are considered in [34]. Methods for analysing how recessions leave curses in their wake in the form of a lower growth-path for outputs, and how booms can leave blessings, in the form of a higher growth-path, are presented in [35] and [36], providing a framework for incorporating Preisach memory-effects into the analysis of macroeconomic systems in general. To date little work has been done on the determination of the rate of inflation in hysteretic systems. An account of output–inflation interaction is needed to fill this gap, requiring a a shift toward vector hysteresis models.

A major problem in conducting empirical tests on Preisach-type models is the lack of information on the Preisach weight function $g(\alpha, \beta)$. In the example of capital investment projects considered earlier there is the inherent problem that the $\alpha$ and $\beta$ trigger values are regarded by firms as being commercially-sensitive information. Otherwise there is a dearth of cross-sectional data on the switching points that putatively allow economic agents to be represented as hysterons. There is also the problem that the switching points could well change over time as agents learn from mistakes or otherwise change their strategies for responding to the economic environment (see [37] for an analysis).

Two of the empirical studies that have been undertaken illustrate both the promise and difficulties associated with testing Preisach-type models in economics. In [38] such a model is used to investigate how hysteresis affected the equilibrium rate of employment in the UK, 1959–1996. The key finding was that, as well as the contemporaneous value of an index of unemployment benefits relative to wages, hysteresis index variables reflecting a selective memory of exchange rates, real oil prices and real interest rate perturbations had significant effects. To generate the hysteresis index variables, the area under the Mayergoyz staircase partition

[11] was approximated as a union of rectangular trapezoids. The Preisach weight function was first specified as a uniform distribution and then sensitivity tests were conducted using normal, Poisson, and exponential distributions. These tests suggested that the alternative distributional assumptions made little difference to the results. A limitation was that the time-series variables contained relatively few peaks and troughs, thus making the results tentative. Higher frequency data, such as is available in relation to financial markets, might permit firmer conclusions to be drawn. In [39] the empirical problem was to explain the way US imports from Japan respond to changes in the dollar–yen exchange rate. The strategy here involved piecewise-linear approximations of macro-hysteresis loops, the slope of the linear functions changing at extremum values. Again the results were positive in that the hysteresis effect was found to be statistically significant, but the empirical method could be at best described as an approximation. To date there has been little work done on the control problems arising in such Preisach models in economics.

It is unfortunate that the most common usage of the term "hysteresis" in economics differs significantly from that defined in this article and employed in the physical sciences. This term is used by many economists to refer in a general way to the persistence of deviations from equilibrium, especially after severe economic shocks [40], [41]. As noted earlier, economists were aware of such macroeconomic persistence effects for a long time, without incorporating them into the mainstream analytical framework. However, eventually an argument consistent with the neoclassical model was forthcoming.

This mainstream explanation of persistence of deviations runs as follows. Suppose that the system can be considered as a linear, discrete time, stochastic difference equation of the form

$$X_t = AX_{t-1} + \eta_t, \qquad X_t \in \mathbb{R}^m, \tag{2}$$

where $\eta_t \ll 1$ is an exogenous stochastic process and $A \in \mathbb{R}^m \times \mathbb{R}^m$. Further, assume that all the eigenvalues of $A$ lie inside the unit circle so that the origin is a stable equilibrium and let $a$ be the largest eigenvalue by magnitude. If some economic shock moves the system away from equilibrium then long transients can be generated if $a \approx 1$ and a continuum of equilibria exist if $a = 1$. Both cases lead to a history-dependent system path over a long time. There is a very large literature on the existence of *unit root processes* with econometric tests claiming to have detected their presence.

The above phenomenological model begs some fundamental questions. Firstly, is there any *a priori* reason why economic equilibria should be close to instability? If so, this then raises the worrying possibility that economic systems are inherently borderline stable with profound implications for the rest of macroeconomics. However this logical consequence of the unit root explanation does not seem to have been considered in the mainstream literature. Secondly, if memory effects are in fact due to the presence of strong hysteresis and remanence, and not the presence of unit roots, how does this affect the statistical tests (for example, the augmented Dickey–Fuller test [42]) for unit roots? It is entirely possible, if not probable, that such econometric tests can be misinterpreted in the presence of hysteresis.

To summarize, suggesting that hysteresis is indeed occurring at microeconomic levels, makes it possible to employ the phenomena of selective memory and remanence to provide a plausible, and relatively well understood, explanation for what is otherwise regarded as "persistence" and "path–dependence" in macroeconomics. Furthermore, hysteresis provides a mechanism by which history dependence and stability can comfortably coexist, in direct contrast to the unit root hypothesis.

## Financial Markets

### *The Efficient Market Hypothesis*

The consequences, both philosophical and practical, of the assumptions underlying the hypothesis of memory-free efficient financial markets cannot be overestimated. Although the concepts were introduced by Bachelier in his 1900 Ph.D. thesis, this work was largely forgotten until the 1960s when the concepts became known collectively as the Efficient Market Hypothesis (EMH) [43], [44], [45]. Firstly, there are strong assumptions about the market itself and the nature of the information stream entering it. This data consists of economic statistics, performance reports, geopolitical events, analysts' projections and so on. It is assumed to be instantly available to all economic participants, uncorrelated with itself, and is usually modelled as a Brownian motion, possibly with drift. A second class of assumptions relate to the market participants themselves, who are deemed to be perfectly rational and capable of instantaneously incorporating new data into their own differing market strategies and predictions. The heterogeneity of agents is necessary to ensure that trading occurs in the absence of arbitrage opportunities (arbitrageurs are agents who can identify and act upon instantaneous riskless profit opportunities due to small market mispricings). Thus the final ingredient in the EMH description is the *rational expectations* assumption that the differing expectations driving trades, when used as predictions, are *on average* correct and do not result in market mispricing. Additional assumptions, such as the absence of transaction costs, yield the standard formulae used for risk management and derivative pricing that form the bedrock of modern financial engineering.

Numerous statistical studies or actual markets and asset prices have shown significant deviations from the implications of the EMH (see [46], [47]) . These differences are surprisingly independent of geography, asset type, trading rules and political systems and have come to be known as the *stylized facts*. We now briefly discuss the two most important such deviations. *Volatility clustering*, also known as *heteroskedasticity*, is the phenomenon whereby the volatility of a financial variable, such as an asset price, varies over time. Volatility clustering is often quantified by measuring the autocorrelation function of the absolute value of the price returns, which decays slowly over several months according to an approximate power law. However, the autocorrelation function of the price returns themselves becomes negligible over a time-scale of several minutes (in almost perfect accordance with the EMH). The typical distribution of the observed price returns (as opposed to their well behaved linear autocorrelation) provides a second major discrepancy. Under the EMH assumptions this distribution should be log-normal, that is, the logarithm of the price returns measured over some constant interval (days, weeks, months, etc) should be Gaussian and thus have exponentially decaying tails. In fact the tails decay much more slowly, obeying an approximate power law, and so the standard EMH models underestimate the frequency of large price changes by many orders of magnitude. These *fat tails* associated with large price changes are often the manifestation of asset price bubbles or the ensuing crashes.

Models assuming constant volatility and log-normal price changes are still routinely used to perform risk analysis and to price financial derivatives, in the celebrated Black–Scholes [48] option pricing formula, for example, despite firm evidence to the contrary. This nonchalance is unnerving to say the least. It is also interesting to note that the volume of financial derivatives transactions exploded almost as soon as the option pricing formula was published in 1973. This interplay between a model and the very system it is trying to describe adds to the points made earlier about the rôle that models play in economics as compared with other disciplines.

*Modelling markets with hysteretic agents*

An immediate consequence of the EMH is that markets have no memory. In other words, all past information is accurately and instantaneously incorporated into the current stock price so that nothing is to be gained by looking at past market data. It says something about the schizophrenic nature of economics and finance that this notion that 'the market is always right' is sometimes upheld by the same people who hire technical analysts or *chartists* to pore over past data to predict future price moves (one possible counterargument is that such technical analysis itself forms part of the pricing mechanism and helps make markets more efficient than they would otherwise be).

We now show how an EMH/rational expectations model involving hysterons can be constructed [49]. Hysterons by definition are history-dependent while the EMH models, by definition, are not. However, the rational expectations framework allows individual agents to have memory dependence provided that, when averaged, this dependence does not manifest itself in the asset price. Consider a system of $M$ agents, each of whom is only able to be long (buy) or short (sell) in one unit of an asset. The (discrete time) system is evolved in time steps of length $h$ and the investment position of the $i^{th}$ investor over the $n^{th}$ time interval is represented by $s_i(n) = \pm 1$ ($+1$ long, $-1$ short). The price of the asset at time $n$ is denoted by $p(n)$ and is subject to an exogenous information stream in the form of a Brownian motion $W(n)$ (note that the time variable $t$ has been scaled so that the variance of $W(t)$ over a unit time interval is 1). An important variable is *sentiment*, defined as the average of the states of all of the $M$ investors

$$\sigma(n) = \frac{1}{M} \sum_{i=1}^{M} s_i(n). \tag{3}$$

We further define $\Delta\sigma(n) = \sigma(n) - \sigma(n-1)$. The price is updated using the formula

$$p(n+1) = p(n) \exp\left(\sqrt{h}\eta(n) - h/2 + \kappa \Delta\sigma(n)\right) \tag{4}$$

where $\kappa \geq 0$ and $\sqrt{h}\Delta W(n) \sim \mathcal{N}(0, h)$ represents the exogenous information stream. If $\kappa = 0$ then the price follows a geometric (driftless) Brownian motion determined only by the external information stream. But when $\kappa > 0$ the price now also depends upon internal dynamics via the market sentiment term reflecting the changing investment positions of the agents.

Each agent is modelled in terms of binary switches. Suppose that at time $n$ the $i^{\text{th}}$ investor has just switched and the current price is $P$. Then a pair of numbers $X_L, X_U > 0$ are generated from some specified distribution (independent of the particular agent) and the lower and upper price thresholds for that agent are set to be $L_i = P/(1 + X_L)$ and $U_i = (1 + X_U)P$ respectively. Thus the agent is considered to be a hysteron who switches instantaneously when either $p(n) > U_i$ or $p(n) < L_i$. When such an event occurs a new set of thresholds straddling the current price is generated. Now suppose that $M$ is large and $\sigma(0) \approx 0$ with the initial states of the agents well mixed. Then the lack of any coupling between agents implies that over any time-step the numbers of agents switching in each direction will cancel and $\sigma$ remains close to 0. Thus the behavior of the system is very close to the case where $\kappa = 0$ and EMH pricing still applies. Further details, economic justifications and numerical simulations can be found in [50], [51], [52], [49].

The above model matches the rational expectations/EMH paradigm, that is, that agents trade because of differing future expectations but the price remains correct because there is no coupling between agents and the differences cancel. However, the threshold values are capable of multiple economic interpretations in addition to the neoclassical one of rational economic analysis. Firstly, the presence of thresholds very naturally incorporates the effects of transaction/sunk costs, exactly as described above for entry-exit problems. Secondly, the psychological pressure to take profits or cut losses (depending upon which threshold is breached) is captured by the hysteron description. Also, experimental economists and psychologists have demonstrated the existence of *anchoring* where investment decisions are strongly influenced by recent experience, in this case the last price at which the asset was traded by that agent. There is now a substantial literature categorizing and attempting to quantify such psychological propensities [53], [54], [55], [56], [57], [58], [59], [60].

As simple as the above model is, it already differs significantly from the standard use of Preisach hysterons in, say, ferromagnetics. In the continuum limit $M \to \infty$ , *the Preisach weight function evolves* as information enters the system, loosely shadowing the price $p(n)$, but the perfect mixing of the hysteron states means that there is no net effect upon the price. However, if coupling between the agents is introduced then the EMH pricing property of the model can be lost, as is now shown.

In [49], building upon previous work [50], [51], [52], a *herding tendency* was introduced into the agents' behavior. The phenomenon of herding has been well documented and appears to be an important factor in most, if not all, financial bubbles. There are several underlying reasons for herding to occur. Firstly there is the psychological propensity for people to feel safer when in the majority, and the positive feedback in the form of momentum trading can mean that yet more people take the same position. Secondly, there are significant (rational but perverse) institutional reasons why professional investors herd into similar market positions. These individuals or their institutions often cannot afford noticeably to underperform the market even for short periods without losing their jobs or investment capital. In [49] the herding effect was modelled by allowing the hysteron thresholds to move between switchings: the thresholds move inwards towards the current price whenever that agent is in the minority, following the rule

$$L_i(n+1) = L_i(n) + C_i h|\sigma(n)|, \quad U_i(n+1) = U_i(n) - C_i h|\sigma(n)|,$$

where the $C_i$ are agent-dependent parameters reflecting the agent's herding propensity. This change means that agents in the minority are now more likely to switch into the majority position than vice versa. The effect of this change was that significant asset "mispricings" occurred, with fat-tailed price returns similar to those observed in real markets. The herding effect did not however induce volatility clustering which was then introduced by additionally supposing that the volume of high-frequency or "noise" trading is correlated with the market sentiment $\sigma$. Note that this ability to infer causal relationships between EMH-violations at the micro-level and non-EMH statistics at the macro-level (at least within this modeling paradigm) arises precisely because the hysteron approach provides a framework within which the EMH assumptions can be replicated and then systematically weakened.

The output of random processes fed through a Preisach filter has been studied in [61], [62]. The above model suggests that financial markets can potentially be viewed the same way, although with the added complication that the Preisach weight function is itself evolving over time. The dynamics and control of such coupled-hysteron systems provides a formidable but fascinating challenge.

## Conclusions and Outlook

This article has considered the use that economists have made of hysteresis concepts, borrowed originally from micro-magnetics and adapted to various contexts of economic activity. Though highly suggestive and intuitively attractive, these concepts have so far had at best an informal influence on economic policy. Their main use has been to frame criticisms of mainstream models that either do not take into account any history dependence or consider only the special case of unit roots in the underlying difference equations. In order that hysteresis, in addition to suggesting an explanatory mechanism, becomes a formal tool in the policy-makers armory, much work remains to be done.

From the theoretical point of view, it is important to characterize the interactions among micro-hysteretic economic agents that do, or do not, lead to macroscopic hysteresis. Another outstanding issue is the rigorous derivation of mean field models in systems with hysteretic miscrostructure [50]. An important empirical task is to ascertain experimentally, in various decision-making contexts, how hysteretic economic agents actually are at the micro-level. Such properties should be studied both in isolation, when the decisions of an agent do not impinge on the economic data that she has to respond to, and in interaction with other agents, in order to understand the types of information used in economic decision making and the nature of the interaction. Detailed surveys are also required to identify the switching points involved in the Preisach weight functions, and how these evolve. Such foundations are required to construct robust models of economic systems with hysteresis, which can then be used to address the important control problems that can arise.

**Sidebar 1: Inflation and Taylor rules**

In a metallic-currency world, the supply of money for monetary use is be determined by gold or silver mining or extraction rates, less the demand for the metals in non-monetary use. Paper money was exchangeable for gold under the Gold Standard, so the total money supply tended to follow that of gold. This policy imparted stability to the price level, the British price level at the start of the 1914-18 World War, for example, being the same as it was fifty years previously. The Bretton Woods international monetary system of 1944-71 involved currencies being pegged to the US dollar, which in turn was exchangeable for gold at $35$$ per ounce. Once the last links to gold were abandoned, attempts to implement the monetarist prescription of controlling inflation by having central banks operate non-feedback rules for the rate of change of the money supply were tried in various countries. By and large these attempts failed, central banks being unable to hit their monetary targets in a world of deregulated financial markets (see [63]). Since the late 1980s the typical monetary control regime has come to be one of independent central banks pursuing inflation targets. This system relies on the “natural rate” hypothesis that $\dot{p} = f(u - u^*) + \dot{p}^e$, where $\dot{p}$ and $\dot{p}^e$ are the actual and expected rates of inflation, and $u$ and $u^*$ are the actual and natural rates of unemployment. For $\dot{p} = \dot{p}^e$, the conditions $u = u^*$ and $f(0) = 0$ need to hold. So for inflation expectations to be consistent with the target rate of inflation, $\dot{p}^*$, central banks need to respond to any emerging discrepancies between $u$ and $u^*$. The Taylor rule,

$$i = a + b(\dot{p} - \dot{p}^*) + c(u^* - u),$$

was proposed to describe how a central bank fixes its input repo rate control variable $i$, the interest rate at which it repurchases qualifying securities from banks in return for cash, in order to hit a $\dot{p}^*$ inflation target [27], with the feedback variables being represented by $(u^* - u)$.

**End of Sidebar 1**

**Sidebar 2: Phillips and his Machine**

Phillips was an electrical engineer who became an economist after his incarceration in a Japanese POW camp, and built an analogue hydromechanical machine to illustrate the workings of Keynesian macroeconomics; see [64] and Figure S1. Phillips went on to write key papers on control, focusing on the problems raised by time lags in the responses of macroeconomic policy targets to variations in input control variables. An important innovation here was the introduction of PID (proportional, integral, derivative) feedback methods to try and design a macroeconomic control system that could correct for shortfalls of GDP in relation to reference values without amplifying the cyclical fluctuations in GDP. In this work Phillips employed a relationship between the rate of inflation and the level of GDP, a forerunner of the estimated curve for which he became famous [65].

**End of Sidebar 2**

**Sidebar 3: Experimental Economics**

A common misconception is that experiments are impossible in economics. There is actually an extensive literature on experimental economics going back to the 1950s, and a currently active research agenda in "neuroeconomics" that exploits findings from neuroscience.

The 2002 Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel was given to the psychologist Kahneman and the economist Smith for their work on experimental economics. What is different in economics is that the experiments are artificial in the sense that the economics laboratory is populated by people who are asked to reveal the choices they would make in hypothetical circumstances that are at a remove from actual economic circumstances. This situation obviously raises the question of the extent to which behavior revealed in the economics laboratory can be translated into situations in the "real" world, an important issue being whether the incentives that can be offered in the laboratory are able to mimic what drives actual economic behavior [66]. At least some of the laboratory-derived results have proved to be robust predictors of actual behavior, in auctions for example. Kahneman's experimental results included those identifying loss aversion, reference dependence and anchoring in the heuristics used to make choices between uncertain prospects [53], [54]. His fellow Nobel laureate Smith's work [55] was concerned, *inter alia*, to see whether experimental markets displayed key neoclassical properties, such as the efficient markets hypothesis claim that market prices reflect all the available information relevant to price determination (see [67] for an illuminating discussion of experimental economics). The Duhem-Quine thesis is particularly relevant here in that laboratory experiments would require controlling for a very large number of auxiliary hypotheses in order to expose a target hypothesis to refutation.

**End of Sidebar 3**

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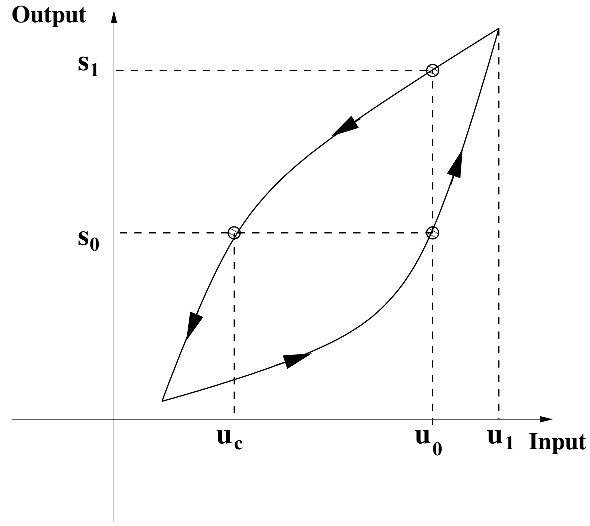


Figure 1. A hysteresis loop, remanence, and coercive force. Here the input increases to $u_1$ and returns to the value $u_0$; the remanence is the difference $s_1 - s_0$, while the magnitude of the coercive force is $u_0 - u_c$.

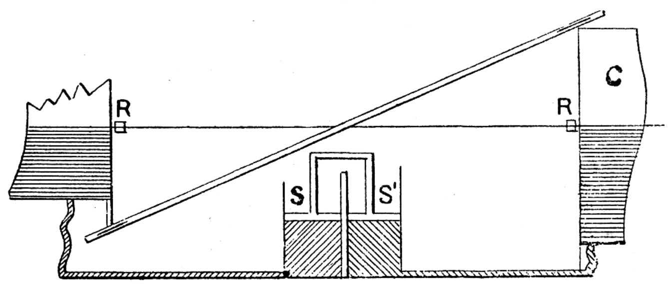


Figure 2. A diagram of Fisher's model. Note how a lever serves to keep marginal utility and marginal cost equal; how duplicate pistons ensure that quantity consumed equals quantity produced; and how the shape of the cisterns depicts the relationships between quantity consumed and marginal utility, and between quantity produced and marginal cost.

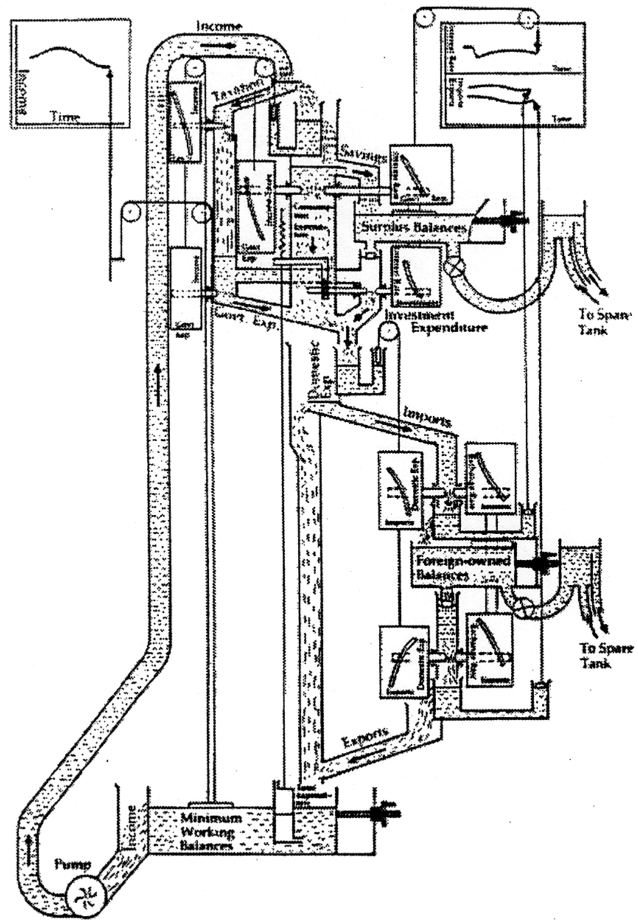


Figure S1. Phillips' machine, which represents the various macroeconomic stocks and flows by colored water flowing through tubes, with mechanical coupling through valves providing feedback from the different parts of the system.