

Solution:

MATH 114, Section 003

Quiz 7

RCT session March 19, 2018.

Write the following improper integral as a limit of a definite integral. Compute the limit and test for convergence.

$$I = \int_3^{\infty} \frac{dx}{x(\log x)^5}$$
$$I = \lim_{b \rightarrow \infty} \int_3^b \frac{dx}{x(\log x)^5} = \lim_{b \rightarrow \infty} \left[-\frac{1}{4} (\log x)^{-4} \right]_3^b$$

$$\int \frac{dx}{x(\log x)^5} \quad \begin{array}{l} u = \log x \\ du = dx/x \end{array}$$
$$= \int \frac{du}{u^5} = \int u^{-5} du = \frac{1}{-5} u^{-5} = -\frac{1}{5} u^{-5} = -\frac{1}{4} u^{-4} = -\frac{1}{4} (\log x)^{-4}$$

$$= \lim_{b \rightarrow \infty} \left(\frac{1}{4} (\log 3)^{-4} - \frac{1}{4} (\log b)^{-4} \right)$$

$$= \lim_{b \rightarrow \infty} \left(\frac{1}{4 (\log 3)^4} - \frac{1}{4 (\log b)^4} \right) = \frac{1}{4 (\log 3)^4}$$

so I is convergent ($I = \frac{1}{4 (\log 3)^4}$)