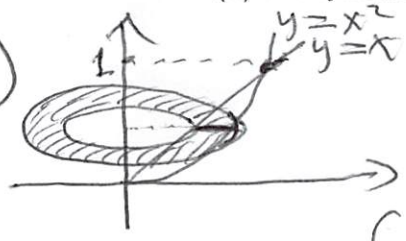
Solution:

MATH 114, Section 003

Quiz 2
RCT session February 5, 2018.

Compute the volume formed when the area between $y = x^2$ and $y = x$ in the first quadrant is revolved around y-axis using (a) circular disc slice method, and (b) the cylindrical shell method.

a)



$x = \sqrt{y}$ $\quad y = \sqrt{y}$ $\quad y(y-1) = 0$

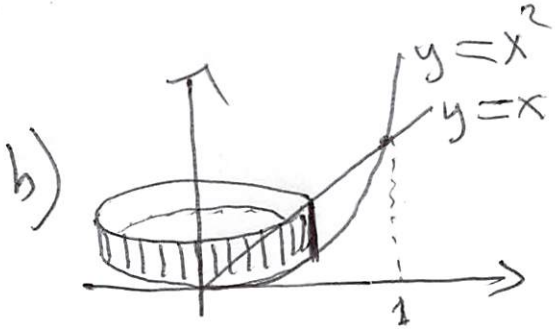
$x = y$ $\quad y^2 = y$ $\quad y = 0, 1$

$$V = \int_0^1 A(y)\,dy = \int_0^1 (\pi r_1^2 - \pi r_2^2)\,dy \qquad \begin{pmatrix} r_1 = \sqrt{y} \\ r_2 = y \end{pmatrix}$$

$$= \pi \int_0^1 ((\sqrt{y})^2 - (y)^2)\,dy = \pi \int_0^1 (y - y^2)\,dy = \pi \left[\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\right]_0^1$$

$$= \pi\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \frac{\pi}{6}.$$

b)



$x^2 = x$ $\quad x = 0, 1$

$x(x-1) = 0$

$$\Rightarrow V = \int_0^1 A(x)\,dx = \int_0^1 2\pi x(x - x^2)\,dx$$

$$= 2\pi \int_0^1 x(x - x^2)\,dx = 2\pi \int_0^1 (x^2 - x^3)\,dx = 2\pi \left[\tfrac{1}{3}x^3 - \tfrac{1}{4}x^4\right]_0^1$$

$$= 2\pi\left(\tfrac{1}{3} - \tfrac{1}{4}\right) = \frac{2\pi}{12} = \frac{\pi}{6}.$$

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