

Solution:

MATH 114, Section 003

Quiz 11

RCT session April 16, 2018.

Test the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{5^n n!}{(2n)!}$$

$$a_n = \frac{5^n n!}{(2n)!} > 0$$

Ratio test: $\frac{a_{n+1}}{a_n} = \frac{\left(\frac{5^{n+1} (n+1)!}{(2n+2)!}\right)}{\left(\frac{5^n n!}{(2n)!}\right)} = \frac{5^{n+1} (n+1)! (2n)!}{5^n \cdot n! (2n+2)!}$

$$= \frac{5(n+1) \cancel{1 \cdot 2 \cdot 3 \cdot \dots (2n)}}{\cancel{1 \cdot 2 \cdot 3 \cdot \dots (2n)} (2n+1)(2n+2)} = \frac{5(n+1)}{(2n+1)(2n+2)}$$

$$= \frac{5(n+1)}{(2n+1)(2n+2)} = \frac{5}{2(2n+1)} \xrightarrow{n \rightarrow \infty} 0 < 1.$$

\Rightarrow Convergent.