MATH 114, Analytic Geometry and Calculus II

Review sheet for first midterm

(1) Use the precise definition of $\log x$ as an integral to prove that for any real a and $x > 0$ we have $\log(x^a) = a \log x$.

(2) Let $1 < a < b$ be real numbers. Find the area bounded by the curves $y = x^a$ and $y = x^b$.

(3) If the area from (2) is rotated about x-axis, find the volume of the resulting solid.

(4) If the area from (2) is rotated about y-axis, find the volume of the resulting solid by integrating in terms of y.

(5) If the area from (2) is rotated about y-axis, find the volume of the resulting solid by integrating in terms of x, using the cylindrical shells.

(6) Prove that a cone with a circular base of radius R and height of h has volume of $\frac{\pi}{3} h R^2$.

(7) Prove that the volume of a solid sphere of radius R is $\frac{4}{3}\pi R^3$. (This is what Archimedes considered one of his most important results!)

(8) An *ellipse* is a curve of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Compute the volumes of the solids obtained by rotating the ellipse about (i) the x-axis, (ii) the y-axis.

(9) Find the arc-length of the curve $y = \log(\cos x)$ from $x = 0$ to $x = \pi/3$.

(10) Find the arc-length of the curve formed by $(x, y) = (\cos^2 t, \sin^2 t)$.

(11) Find the center of mass of the uniform plate formed by the upper half of the ellipse from (10).

(12) For $a \in [0; \pi]$, compute the center of mass CM_a of a uniform string $(x, y) = (R \cos t, R \sin t)$, where t ranges from $-a$ to a. Compute the limits

$$\lim_{a \to 0} CM_a \text{ and } \lim_{a \to \pi} CM_a.$$

(13) Use the definition of $\sinh x$ to compute the formula for $\sinh^{-1} x$. For what real x is this formula valid?

(14) Compute the following limit:

$$\lim_{x\to\infty} \frac{\sinh^{-1} x}{\log x}.$$

(15) Compute the following integrals:

$$\int \frac{(x+2)dx}{x^2-1}, \quad \int \frac{(x+2)dx}{x^2+1}, \int \frac{(x^3+2)dx}{x^2+1}, \int \frac{(x+2)dx}{x^3-1}.$$

(16) Compute the partial fraction form of the following rational functions

$$\frac{1}{(x-1)(x-2)(x-3)(x-4)},$$

$$\frac{x^4}{(x^2-1)(x^2-2)(x^2-3)(x^2-4)},$$

$$\frac{x^8}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}.$$

NOTE! This is not a recipe for the midterm exam!

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