Department of Mathematical Sciences George Mason University

Math 114, Analytic Geometry and Calculus II

Second MATHEMATICA assignment

You may work on this assignment in groups containing up to 3 people. If you work in a group, please turn in just one copy of the assignment with all your names clearly printed and signed on it. – Be sure to answer the questions posed in complete sentences and in an organized manner! Do not turn in a MATHEMATICA output without accompanying explanations. Your assignment you turn in should read as a report of a specific investigation and should therefore be consistent and self-contained!

DUE DATE: April 30, 2018, in your RCT session.

(1) (50%)
In class we have discussed the *Pickard's Method* for solving heuristically with an arbitrary precision an equation of the form

$$g(x) = x, \tag{1}$$

where $g(x)$ is a continuous function and where it is impossible to solve (1) analytically. This method is based on obtaining a convergent sequence $(x_n)_{n=0}^{\infty}$ defined recursively by

$$\begin{aligned} x_0 &= \text{ some well chosen number },\\ x_{n+1} &= g(x_n) \text{ for } n \geq 0, \end{aligned}$$

where $\lim_{n\to\infty} x_n = r$ where r is the solution to (1), that is $g(r) = r$.

Consider the equation

$$e^x = x + 5. \tag{2}$$

In what follows, we would like to compute the positive solution (2).

1. (10%) Plot the graphs $y = x$ and $y = e^x - 5$ in the same coordinate system. How many solutions does (2) seem to have? How many positive solutions and how many negative?
2. (15%) Use MATHEMATICA where $g(x) = e^x - 5$ to obtain the recursive defined sequence $(x_n)_{n=0}^{\infty}$ with $x_0 = 2$. Compute the first five terms of this sequence with 30 correct decimals. Does the sequence seem to converge? What if you start with $x_0 = 0$ instead?

3. (15%) Use MATHEMATICA where now $g(x) = \ln(x+5)$ to obtain the recursively defined sequence $(x_n)_{n=0}^{\infty}$ with $x_0 = 0$. compute the first few terms of this sequence. Does the sequence seem to converge? If so, compute the limit with 30 decimal precision. Is this limit a solution to (2)?

4. (10%) Give a reason as to why the second method worked to compute the positive solution to (2). (No MATHEMATICA is needed here!) (*Hint:* Look into your class notes.)

(2) (50%)
Consider the series

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)(n+3)} = \frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{3\cdot 4\cdot 5} + \cdots. \qquad (3)$$

1. (10%) Use MATHEMATICA to compute the first k partial sums of the above series in (3) where $k = 10^i$ and $i = 2, 3, 4, 5, 6, 7, 8, 9$ and 10. Does it seem to converge? What does the limit appear to be?

2. (10%) Use MATHEMATICA to evaluate the total sum of the series.

3. (10%) Compare with a p-series, and prove that the series in (3) is convergent. (No MATHEMATICA is needed here!)

Now consider the series

$$\sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \cdots. \qquad (4)$$

1. (10%) Use MATHEMATICA to compute the first k partial sums of the above series in (4) where $k = 10, 20, 30, 40$ and 50. Does it seem to converge? Do you recognize the limit?

2. (10%) Use MATHEMATICA to evaluate the total sum of the series in (4).

Turn your your solutions to your TA by the mentioned due date.

Geir Agnarsson