Department of Mathematical Sciences George Mason University

Math 114, Analytic Geometry and Calculus II

First MATHEMATICA assignment

You may work on this assignment in groups containing up to 3 people. If you work in a group, please turn in just one copy of the assignment with all your names clearly printed and signed on it. – Be sure to answer the questions posed in complete sentences and in an organized manner! Do not turn in a MATHEMATICA output without accompanying explanations. Your assignment you turn in should read as a report of a specific investigation and should therefore be consistent and self-contained!

DUE DATE: April 9, 2018, in your RCT session.

(1) (50%)
We have discussed in class the fact that differentiation is *easier* than integration in the following sense: If a function of x is given as a composition of the usual functions in our tool bag, like x^a, $\log x$, e^x, $\cos x$, $\sin x$ and such, then you are in principle always able to differentiate this function in a systematic manner. However, this is not the case for integration, since there are easily stated integrals that cannot be evaluated in terms of our tool bag of functions. Some examples of integrals that *cannot* be evaluated are as follows:

$$\int e^{x^2} dx, \quad \int \frac{e^x}{x} dx, \quad \int \frac{dx}{\log x},$$
$$\int \sin(x^2) dx, \quad \int \frac{\sin x}{x} dx, \quad \int \sin(e^x) dx.$$

1. (30%) Using MATHEMATICA, evaluate the following indefinite integrals.
$$\int x^{23} e^{x^4} dx, \quad \int x^{24} e^{x^4} dx,$$
$$\int x^{25} e^{x^4} dx, \quad \int x^{26} e^{x^4} dx.$$
Make MATHEMATICA simplify the analytic (precise) answer it provides in each case. You will see that sometimes the answer it gives is in terms of functions that we know, other times it expresses its answer

in strange functions that cannot be written in terms of functions from our tool bag.

2. (10%) Prove that for each $n \in \mathbb{N}$ we have

$$\int x^n e^{x^4} dx = \frac{x^{n-3}e^{x^4}}{4} - \frac{n-3}{4}\int x^{n-4}e^{x^4} dx.$$

   (*Hint:* Differentiate both sides and verify that you get the same answer. No MATHEMATICA is need for this part!) This shows that if you can evaluate the integral for $n-4$, then you can also evaluate it for n and vice versa.

3. (10%) Using the outcome of the first part and then the general property from the second part, what do you conjecture about the solubility of $\int x^n e^{x^4} dx$ for an arbitrary natural number $n \in \mathbb{N}$?

(2) (50%)
Let $n \in \mathbb{N}$ be a fixed natural number. Consider the parametrized curve given by (x, y) where

$$\begin{aligned} x(t) &= \sin(nt)\cos(t), \\ y(t) &= \sin(nt)\sin(t). \end{aligned}$$

and t ranges from 0 to 2π. The resulting curve is called an *n-rose* R_n.

1. (10%) Show that R_1 is a circle with center at $(0, 1/2)$ and radius $1/2$ (No MATHEMATICA is needed for this part).

2. (20%) For $n = 1, 2, 3, 4, 5, 6$ plot the n-roses R_n.

3. (10%) From the plotted curves in (2), what do you conjecture about the number of leaves each n-rose has?

4. (10%) Use MATHEMATICA to compute the arc length of R_3 with 30 decimals.

Turn your your solutions to your TA by the mentioned due date.

Geir Agnarsson