Algebraic Methods for Faster Enumeration of Unsensed Planar Maps

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Abstract

Nick's papers of some 30 years ago (*Canad. J. Math.* **33** (1981) 1–11 and *Discrete Math.* **36** (1981) 205–225) developed a method of counting unsensed planar maps. He derived a basic set of nonlinear differential equations satisfied by the generating functions for counting maps which are rooted in various ways.

The numbers of unsensed maps are then found by combining the rooted map numbers using a form of *Burnside's Lemma.*

Recursive comparison of coefficients in the basic equations gives a straightforward method of finding the numbers of unsensed maps with up to N edges in time $O(N^4)$ by edges only, and in $O(N^6)$ time by edges and vertices.

The present work shows the existence of a way to compute those numbers in time $O(N \log N)$ by edges only, and time $O(N^2)$ by edges and vertices.

The idea is to apply the kernel method to find simultaneous algebraic equations for some of the basic types of rooted maps, then the theory of multivariate *D-finite* (or *holonomic*) series to deduce that the numbers of the various types of rooted maps can be calculated in constant amortized time.

Combining the rooted map numbers to find the numbers of unsensed maps introduces the extra factor of $O(\log N)$ to the total time for counting by edges only.