Workshop on Combinatorics, Enumeration, and Invariant Theory (WCEIT)[1]

March 19 – 20, 2010

George Mason University, Fairfax, Virginia

Abstracts of Invited Talks

Plenary Talks by Vic Reiner:

Cyclic actions and invariant theory

Victor Reiner
School of Mathematics
University of Minnesota
Minneapolis, MN – 55455
reiner@math.umn.edu

Abstract

In previous work with Dennis Stanton and Dennis White, we defined the "cyclic sieving phenomenon", whereby one has a cyclic action on a set, and an associated generating function that encodes the information about this action in an amazingly simple way. – After reviewing two examples of this phenomenon, we show how they led us to a result in the (characteristic-free) invariant theory of finite groups. This is joint work with A. Broer, L. Smith and P. Webb.

When two cyclic groups commute: the bicyclic phenomenon

Victor Reiner
School of Mathematics
University of Minnesota
Minneapolis, MN – 55455
reiner@math.umn.edu

Abstract

We discuss some of our favorite examples of a phenomenon that can occur when one has commuting actions of *two* cyclic groups on a finite set. This is joint work with A. Berget, S.-P. Eu and D. Stanton.

---

[1]Supported in part by a grant from the Office of Naval Research (ONR) http://www.onr.navy.mil/

Complementary Talks:

Cyclic Flats, Sticky Matroids, and Intertwines

Joseph E. Bonin
Department of Mathematics
The George Washington University
Washington, DC – 20052
jbonin@gwu.edu

Abstract

There are many perspectives that one can take on matroids. This talk uses recent progress on two problems to illustrate the merits of the approach to matroid theory via cyclic flats and their ranks, which was developed independently by Julie Sims and by J. Bonin and Anna de Mier. – The first problem we consider is the sticky matroid conjecture. Given a matroid, can each pair of extensions by disjoint sets be 'glued together'? Matroids with this property are called sticky. It is conjectured that sticky matroids are precisely modular matroids (which are direct sums of projective geometries). For a wide class of non-modular matroids, we use cyclic flats to construct pairs of extensions that contain incompatible geometric structures and so cannot be glued together, thus showing that these non-modular matroids are not sticky. – The second problem is that of intertwines. An intertwine of a pair of matroids (or graphs) is a matroid (or graph) such that it, but none of its proper minors, has minors that are isomorphic to each matroid (or graph) in the pair. Intertwines arise naturally when considering the unions of minor-closed classes of matroids or graphs. A corollary of Robertson and Seymour's graph minors theorem is that any pair of graphs has only finitely many intertwines. We show that the situation is very different for matroids: we use cyclic flats to construct infinite sets of intertwines for a broad class of pairs of matroids. – Sufficient background in matroid theory will be provided to make this talk accessible.

Distribution of some combinatorial statistics on old and new Dumont permutations

Alexander Burstein
Department of Mathematics
Howard University,
Washington, DC – 20059
aburstein@howard.edu

Abstract

We will define two new types of Dumont permutations in addition to the two previously known types and show that some combinatorial statistics have the same distribution on old and new Dumont permutations. In addition, we will find statistics on old and new Dumont permutations that generate the Seidel triangle for Genocchi numbers. This is joint work with Matthieu Josuat-Vergs and Walter Stromquist.

Art Galleries and Roller Coasters

T. S. Michael
Mathematics Department
U. S. Naval Academy
Annapolis, MD – 21402
tsm@usna.edu

Abstract

There are two parts of this talk. – Art gallery problems ask for the minimum number of guards needed to protect a given geometric object. We emphasize results obtained using coloring arguments. – The Roller Coaster Conjecture specifies the up-down behavior of the independence sequence $c_0, c_1, \ldots$, for well-covered graphs. Here c_k is the number of independent k-sets of vertices in the graph.

Lattice point methods for combinatorial games

Ezra Miller
Mathematics Department
Duke University, Box 90320
Durham, NC – 27708-0320
ezra@math.duke.edu

Abstract

Dawson's chess is a disarmingly simple two-player game in which each player's unfortunate goal is to lose—that is, to force the other to move last. Strategies for such *misère* combinatorial games are remarkably hard to come by, and even Dawson's simple chess puzzle has no known efficient strategy, after more than 75 years. The aim of this talk is to reformulate the theory of combinatorial games—things like Nim, Checkers, or Chess, in which two players alternate taking turns with complete information and no elements of chance—in terms of lattice points in polyhedra. The point is to reveal periodicities and encode game-theoretic notions, such as winning strategies, via generating functions. With luck (i.e., conjecturally), these encodings can be computed in polynomial time. This work is joint with Mike Weimerskirch and Alan Guo (arXiv:math.CO/0908.3473).

Submagmas of Random Magmas

Keith A Pardue
National Security Agency (NSA)
Fort George G. Meade, MD – 20755
kapardu@nsa.gov

Abstract

A *magma* is a set M with a binary operation $M \times M \to M$. It need not satisfy any additional axioms, such as associativity. If M is a finite set and a binary operation is chosen uniformly at random, then how many submagmas does M have? The asymptotic distribution of the number of submagmas is surprisingly simple, and the expected number of submagmas goes rapidly to 3.

Coarsening polyhedral complexes

Nathan Reading
Department of Mathematics
North Carolina State University,
Raleigh, NC – 27695
nreadin@ncsu.edu

Abstract

A polyhedron is an intersection of finitely many halfspaces. A polyhedral complex is a finite collection of polyhedra which intersect "nicely." A polyhedral complex is complete if the union of all of its polyhedra is the entire ambient space. A polyhedral complex C' is said to coarsen another complex C if every polyhedron in C' is a union of polyhedra in C. – I will describe a local codimension-2 criterion which characterizes coarsenings of a complete polyhedral complex. The criterion broadly generalizes a result of Morton, Pachter, Shiu, Sturmfels and Wienand, which identifies the "semigraphoids" of nonparametric statistics with coarsenings of the polyhedral complex defined by the braid arrangement. The criterion is also closely related to my earlier work on fans defined by lattice congruences of the weak order. I will also sketch the proof, which makes use of a generalization of Tits' solution to the Word Problem and a surprising shortcut for checking whether a set of polyhedra is a complete fan.

Algebraic Methods for Faster Enumeration of Unrooted Planar Maps

Robert W. Robinson
Department of Computer Science
The University of Georgia
415 Graduate Studies Research Center
Athens, GA – 30602
rwr@cs.uga.edu

Abstract

This is joint work with N. C. Wormald, whose papers of some 30 years ago (*Canad. J. Math.* **33** (1981) 1–11 and *Discrete Math.* **36** (1981) 205–225) presented a polynomial time method of counting unrooted planar maps. In those papers he derived a basic set of nonlinear differential equations satisfied by the generating functions for counting maps which are rooted in various ways. With suitable boundary conditions these basic equations determine all of the counting series for the various types of rooted maps. The numbers of unrooted maps with specified numbers of edges (or edges and vertices) are found by combining the rooted map numbers using a form of Burnside's Lemma. Recursive comparison of coefficients from the basic equations gives a straightforward method of finding the numbers of unrooted maps with up to N edges in time $O(N^6)$. – The present work shows the existence of a way to compute those numbers in time $O(N \log N)$. The idea is to apply the kernel method to the basic equations so as to produce a set of simultaneous algebraic equations for the generating functions of the needed types of rooted maps. It is then shown that each of the generating functions individually is algebraic. This implies that each of the rooted map counting sequences satisfes a recurrence of finite length. We do not find these recurrences explicitly, but their existence shows that the numbers of the various types of rooted maps can be calculated in constant amortized time. When these values are combined to give the numbers of unrooted maps by edges up to N this takes $O(N \log N)$ for the total time, and $O(N^2 \log N)$ time for counting by numbers of edges and vertices.

Combinatorial and colorful proofs of cyclic sieving phenomena

Bruce Sagan
Department of Mathematics/Division of Mathematical Sciences
Michigan State University/National Science Foundation)
East Lansing, MI – 48824/Arlington, VA – 22230
sagan@math.msu.edu/bsagan@nsf.gov

Abstract

Let S be a set which admits an action of the cyclic group C_n of order n. Let $f(q)$ be a polynomial in q. Usually $f(q)$ will be the generating function for some statistic on S. Finally, let g be a generator of C_n and $\gamma = e^{2\pi i/n}$. We say that the triple $(S, C_n, f(q))$ exhibits the *cyclic sieving phenomenon (CSP)* if, for every integer d,

$$f(\gamma^d) = \text{ number of element of } S \text{ fixed by } g^d.$$

This concept was first introduced and studied by Reiner, Stanton and White as a generalization of Stembridge's $q = -1$ phenomenon which is the case $\#G = 2$. – Most proofs that a pair exhibits the CSP use either algebraic manipulations involving roots of unity or representation theory. We will present the first (to our knowledge) completely combinatorial proof of a CSP. We will also discuss colored versions of some known examples of the CSP. This is joint work with Yuval Roichman.

Some new applications and theory of the Riordan group

Louis Shapiro
Department of Mathematics
Howard University,
Washington, DC – 20059
lshapiro@howard.edu

Abstract

The Riordan group is useful for giving quick proofs for various binomial identities and for inverting them systematically by providing some group structure. After recalling these basics we will discuss the Double Riordan group and some new examples such as summer winter trees and some other examples involving ordered trees with a mutator vertex. This gives interesting settings for some classical sequences such as the Catalan, Motzkin, and Schroder numbers as well as many new related sequences.

Identifiable Gaussian Graphical Models

Seth Sullivant
Department of Mathematics
North Carolina State University
Raleigh, NC – 27695
smsulli2@ncsu.edu

Abstract

A Gaussian graphical model is a statistical model that uses a graph to encode linear relationships between a collection of random variables. These linear relationships determine a positive definite covariance matrix whose entries are polynomials in certain parameters associated to the edges of the graph. The edge parameters are identifiable if they can be recovered from the covariance matrix. I will discuss recent results, where we employ techniques from computational algebra and algebraic combinatorics to make progress on the identifiability problem. This is joint work with Mathias Drton, Rina Foygel, Luis Garcia, and Sarah Spiegelvogel.