Ideals of skew-symmetric matrix Schubert varieties

Brendan Pawlowski
University of Southern California, Los Angeles, CA – 90089

Abstract

We consider the set of skew-symmetric matrices whose upper-left $i \times j$ corner has rank at most $r_{ij}$, for some integer matrix $r$ and all $i, j$. Such a set is defined by the vanishing of a collection of Pfaffians of submatrices, but it is not obvious that these Pfaffians generate a prime ideal, and they do not usually make up a Gröbner basis for the ideal they generate (with respect to reverse lex order). We prove that these ideals are prime (assuming a simple combinatorial condition on $r$), and show how to extend the Pfaffian generating set to a Gröbner basis. We also show that their initial ideals are squarefree, describe the associated Stanley-Reisner complexes in terms of combinatorial objects called skew-symmetric involution pipe dreams, and prove that these complexes are shellable. Based on joint work with Eric Marberg.

Keywords: Pfaffian, prime ideal, Gröbner basis, Stanley-Reisner complex, shellability.