Routing numbers of some graphs

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Abstract

Let $G$ be a graph whose vertices are labeled 1, \ldots, $n$, and $\pi$ be a permutation on $[n] := \{1, 2, \ldots, n\}$. A pebble $p_i$ that is initially placed at the vertex $i$ has destination $\pi(i)$ for each $i \in [n]$. At each step, we choose a matching and swap the two pebbles on each of the edges. Let $rt(G, \pi)$, the routing number for $\pi$, be the minimum number of steps necessary for the pebbles to reach their destinations.

Li, Lu, and Yang proved that $rt(C_n, \pi) \leq n - 1$ for every permutation $\pi$ on the $n$-cycle $C_n$ and conjectured that for $n \geq 5$, if $rt(C_n, \pi) = n - 1$, then $\pi = 23\cdots n1$ or its inverse. By a computer search, they showed that the conjecture holds for $n < 8$. In this talk, we will outline a proof of the conjecture. We will also talk about the $rt(P_n, \pi)$ when $\pi$ has a given bandwidth, where $P_n$ is a path of $n$ vertices.

Keywords: vertex labeling, graph pebbling, routing number.