Routing numbers of some graphs

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Abstract

Let G be a graph whose vertices are labeled $1, \ldots, n$, and π be a permutation on $[n] := \{1, 2, \cdots, n\}$. A pebble p_i that is initially placed at the vertex i has destination $\pi(i)$ for each $i \in [n]$. At each step, we choose a matching and swap the two pebbles on each of the edges. Let $rt(G, \pi)$, the routing number for π, be the minimum number of steps necessary for the pebbles to reach their destinations.

Li, Lu, and Yang proved that $rt(C_n, \pi) \leq n - 1$ for every permutation π on the n-cycle C_n and conjectured that for $n \geq 5$, if $rt(C_n, \pi) = n - 1$, then $\pi = 23 \cdots n1$ or its inverse. By a computer search, they showed that the conjecture holds for $n < 8$. In this talk, we will outline a proof of the conjecture. We will also talk about the $rt(P_n, \pi)$ when π has a given bandwidth, where P_n is a path of n vertices.

Keywords: vertex labeling, graph pebbling, routing number.