Acyclic digraphs giving rise to complete intersections

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Abstract

The columns of the node-arc incidence matrix of a connected acyclic directed graph $D$ generate an affine semigroup under addition. If $D$ has $m$ nodes and $n$ arcs, then the nullspace of the node-arc incidence matrix has dimension $n - m + 1$.

The Laurent monomials $t_i t_j^{-1}$ corresponding to arcs $(i, j)$ of $D$ generate a multiplicative semigroup $S_D$. The kernel of the homomorphism

$$
\phi : k[u_1, \ldots, u_n] \rightarrow k[t_1, \ldots, t_m, t_1^{-1}, \ldots, t_m^{-1}],
$$

where $\phi(u_r) = t_i t_j^{-1}$ if $(i, j)$ is the $r^{th}$ arc of $D$, is called the toric ideal $I_D$. It is known that $I_D$ is generated by binomials (differences of monomials), and that a minimal generating set contains at least $n - m + 1$ binomials. We characterize connected acyclic digraphs for which $I_D$ is generated by exactly $n - m + 1$ binomials.

Keywords: acyclic directed graph, incidence matrix, affine semigroup, Laurent monomials, toric ideal.