Acyclic digraphs giving rise to complete intersections

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Abstract

The columns of the node-arc incidence matrix of a connected acyclic directed graph D generate an affine semigroup under addition. If D has m nodes and n arcs, then the nullspace of the node-arc incidence matrix has dimension $n - m + 1$.

The Laurent monomials $t_i t_j^{-1}$ corresponding to arcs (i, j) of D generate a multiplicative semigroup S_D. The kernel of the homomorphism

$$\phi : k[u_1, \ldots, u_n] \to k[t_1, \ldots, t_m, t_1^{-1}, \ldots, t_m^{-1}],$$

where $\phi(u_r) = t_i t_j^{-1}$ if (i, j) is the r^{th} arc of D, is called the *toric ideal* I_D. It is known that I_D is generated by binomials (differences of monomials), and that a minimal generating set contains at least $n - m + 1$ binomials. We characterize connected acyclic digraphs for which I_D is generated by exactly $n - m + 1$ binomials.

Keywords: acyclic directe graph, incidence matrix, affine semigroup, Laurent monomials, toric ideal.