

Conjugation of Transitive Permutation Pairs and Dessins d'Enfants

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Abstract

Suppose x and y are permutations which, together, generate a transitive subgroup. For which s is it true that x and the s -conjugate of y also generate a transitive subgroup? Such transitive permutation pairs encode Dessins d'Enfants: a dessin d'enfant (D, X) is a closed oriented surface X with an embedded bicolored graph D whose complement $X \setminus D$ is homeomorphic to a union of discs. Dessins d'enfants are known to have great arithmetic significance, and the Absolute Galois Group permutes them in a very mysterious way. The permutation pairs of two dessins d'enfants that share certain elementary combinatorial features are related by conjugations as above, and dessins d'enfants in the same Galois-orbit share these features and more. I answer the above question for the fundamental case that s is a transposition, and also classify transpositions s according to how they change the genus of X . Central to this is a certain classification of the set of all (x, y) that is essentially about Pattern Avoidance, and descriptions of the same flavor appear in all other classifications. Some of the tools may have use beyond Dessins d'Enfants.

Keywords: Permutation, transitive group, oriented surface, Dessins d'Enfants surface.