Distance magic and group distance magic graphs

Dalibor Froncek, University of Minnesota Duluth, Duluth, MN – 55812

Abstract

Let $G$ be a graph with $n$ vertices and $f$ a bijection $f : V(G) \rightarrow \{1, 2, \ldots, n\}$. We define the weight of vertex $x$ as the sum of the labels of its neighbors, that is,

$$w(x) = \sum_{xy \in E(G)} f(y).$$

When all vertices have the same weight, say $w(x) = m$ for every $x$, then $f$ is called distance magic labeling and $G$ is a distance magic graph.

At IWOGL 2010, Arumugam presented a list of open problems on distance magic labelings. We present solutions to some of them as well as some other recent results.

However, it turns out that this type of magic labeling is very restrictive and consequently even many classes of vertex transitive graphs are not distance magic.

As an example, we prove that for $d \equiv 0, 1, 3 \pmod{4}$ the hypercube $Q_d$ with $2^d$ vertices is not distance magic. On the other hand, we disprove a conjecture by Acharya, Rao, Singh and Parameswaran, who believed that hypercubes are not distance magic except $Q_2$ and present a distance magic labeling for $Q_6$. This was recently generalized by Gregor and Kovar who found a distance magic labeling for of $Q_d$ for any $d \equiv 2 \pmod{4}$.

Such negative results then rise a question whether it would not be more natural to perform the addition in $\mathbb{Z}_n$ rather than in $\mathbb{Z}$. Graphs that satisfy the above definition with the provision that the addition is performed in $\mathbb{Z}_n$ will be called $\mathbb{Z}_n$-distance magic.

To support this idea, we show some examples of graphs that are not distance magic yet are $\mathbb{Z}_n$-distance magic. We show that when we perform addition $\mathbb{Z}_{2^d}$ rather than in $\mathbb{Z}$, then $Q_d$ is $\mathbb{Z}_{2^d}$-distance magic if and only if $d$ is even.

We present some results on $\Gamma$-distance magic labelings of products of cycles and pose several open problems.

Keywords: Distance magic labeling, group distance magic labeling, graph product, hypercubes.