A directed Steinitz theorem for oriented matroid programming

Walter Morris, George Mason University, Fairfax VA – 22030

Abstract

Holt and Klee proved that if $P$ is a $d$-dimensional polytope and $f$ is a linear function on $P$ that is not constant on any edge of $P$, there are $d$ independent monotone paths from the source to the sink of the digraph defined by the vertices and edges of $P$ directed according to the directions of increase of $f$. Mihalisin and Klee proved that every orientation of the graph of a 3-polytope that is acyclic and admits 3 independent monotone paths from the source to the sink is obtained from some 3-polytope $P$ and some linear function $f$ on $P$. We prove analogs of Mihalisin and Klee’s theorem and the 3 and 4-dimensional versions of Holt and Klee’s theorem for oriented matroid programs. Here acyclicity is replaced by the requirement that there be no directed cycle contained in a face of the polytope.

Keywords: polytope, directed path, source, sink.